

Evaluation of the maximized power of a regenerative endoreversible stirling cycle using the thermodynamic analysis

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Abstract: In this paper, the optimal power on endoreversible Stirling cycle with perfect regeneration has been investigated. In the endoreversible cycle, external heat transfer processes are irreversible processes. Optimal temperature of the heat source leading to a maximum power for the cycle is detained. Moreover, effect of design parameters of the Stirling engine on the maximized power of the engine and its corresponding thermal efficiency is studied.

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Keywords: ideal regenerative, Stirling engine, power, volume ratio, thermal efficiency

1. Introduction

In recent years, the main researches and developments have been placed in the electricity production and pollutant reduction especially carbon dioxide. The Stirling engine has a high potential to be applied for converting heat into the mechanical work with a high thermal efficiency. In theory its thermal efficiency might be as high as the Carnot efficiency when it is ideal regenerator, reversible, and having isothermal compression and expansion processes. Stirling engine is an external combustion engine, so it can be powered by various heat sources and waste heat [1-8]. Blank et al.[4] studied the power optimization of an endoreversible Stirling cycle and provided an estimate of potential performance for a real engine. Thombare and, Verma [5] gathered the available technologies and obtained achievements with regard to the analysis of Stirling engines and, at the end, presented some suggestions for their applications. Formosa and Despesse [7] conducted the modeling by means of the isotherm model in order to investigate the effects of dead volumes on the engine's output power and efficiency.

The thermal efficiency of Stirling engines reaches to 40% while the efficiency of Otto and Diesel engines are 25% and 35% respectively. The Stirling engine cycle is a closed regenerative thermodynamic cycle, with cyclic compression and expansion of the working fluid at different temperature level [9-19]. engines are Tlili *et al.* [17] and Martaj *et al.* [18], among others. Tlili *et al.* [17] developed a first law model with additional

consideration for an internal irreversibility parameter to examine the effects of irreversibility on net work, heat addition, and thermal efficiency. Their irreversibility parameter originates from the second law of thermodynamics for a real cycle. Their results regarding dead volume and regeneration effects support the observations of Kongtragool and Wongwises [19]. They also reported that larger irreversibilities in the Stirling cycle thermodynamic processes lead to smaller maximum net work output and thermal efficiency. However, they did not quantify the roles of the heat exchangers, regenerator, and dead volume irreversibilities on the overall system irreversibility.

The development of finite-time thermodynamics [20-24], a new discipline in modern thermodynamics, provides a powerful tool for performance analysis of practical engineering cycles. Several authors have studied the finite-time thermodynamic performance of the Stirling engine [23-30]. Many investigators [4, 23-30] have studied the effect of heat transfer on the power output of a Stirling engine. Petrescu [23, 25] obtained an optimal efficiency for maximum power output of a solar Stirling engine with imperfect regeneration. In addition, finite-time thermodynamics analysis of heat engines is usually restricted to systems having either linear heat transfer law dependence to the temperature differential both the reservoirs and engine working fluids [6, 28-34]. Li et al developed a mathematical model for the overall thermal efficiency of solar powered high temperature differential dish Stirling engine with

finite heat transfer and irreversibility of regenerator and optimized the absorber temperature and corresponding thermal efficiency [26]. Tlili investigated effects of regenerating effectiveness and heat capacitance rate of external fluids in heat source/sink at maximum power and efficiency [28]. Kaushik et al studied the effects of irreversibilities of regeneration and heat transfer of heat/sink sources [30].

In real mode on Stirling engines, heat regeneration has to be considered imperfect. So the optimal power theory design of the Stirling engine will be complex. By assuming an ideal Stirling engine, can be reach to an acceptable result in power output and thermal efficiency. This article has focused on this sector.

2. Thermodynamic Modeling

The regenerative endoreversible Stirling cycle is depicted in Fig.1 and 2. Stirling cycle has two irreversible isothermal processes and two reversible isobar processes. External heat transfer processes are carried out in finite time. Process 1-2 is performed at the constant temperature T_h and process 3-4 is performed at the constant temperature T_c . In this work it is assumed that a perfect regeneration is occurred in the regenerator.

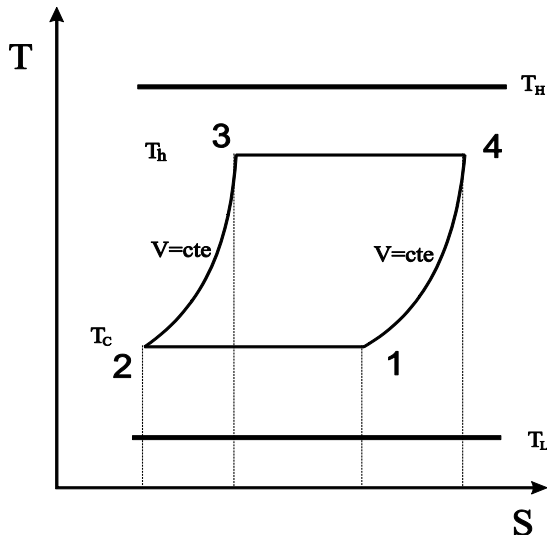


Fig.1. Temperature-entropy diagram of the regenerative endoreversible stirling engine.

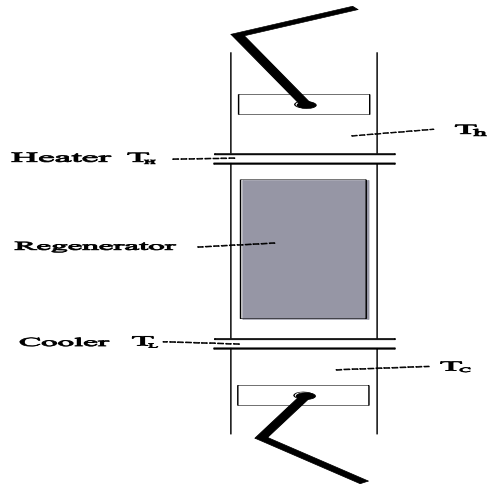


Fig.2. Functional schematic of stirling engine(note that the opposing pistons are 90 degrees out of phase).

At a perfect regeneration, the amount of rejected heat from the hot working fluid to the regenerator mesh during process 1-4 is equal to the amount of absorbed heat by the cold working fluid during process 2-3. This assumption is acceptable, as regenerators with effectiveness of 98% and 99% have been reported [1,4]. Therefore, we have;

$$|Q_{2-3}| = |Q_{4-1}| \tag{1}$$

So it can be shown that the net work of the cycle is obtained as follow [1,4,26-30];

$$W_{net} = mR(T_h - T_c) \ln(\lambda) \tag{2}$$

where m is the mass of the working fluid, R is the gas constant, T is the temperature of working fluid in each state of the cycle as described by subscripts and λ is the compression ratio. The output power is obtained by dividing the net work to the time period of the cycle [1,4,26-30];

$$P = \frac{W_{net}}{t} = \frac{mR(T_h - T_c) \ln(\lambda)}{t_h + t_c + t_{reg}} \tag{3}$$

where t_R is the total time spent for the regeneration processes and t_h and t_c are the times spent for isothermal expansion and compression processes respectively. These time periods are defined as follows;

$$t_h = \frac{mRT_h \ln(\lambda)}{U_H A_H (T_H - T_h)} \tag{4}$$

$$t_c = \frac{mRT_c \ln(\lambda)}{U_L A_L (T_c - T_L)} \tag{5}$$

$$t_R = \frac{2m c_v (T_h - T_c)}{U_{reg} A_{reg} (LMID)_{reg}} \quad (6)$$

where U_H and A_H are the overall heat transfer coefficient and the heat transfer surface area for the hot side heat exchanger, respectively. Similarly, U_L , A_L and U_{reg} , A_{reg} , similar values for cold side heat exchanger and regenerator. T_H and T_L are the temperature of heat source and heat sink respectively. c_v is the specific heat at constant volume in $\text{kJ}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$ and is assumed to be constant.

Assuming the ideal regenerative, $(LMID)_{reg} = 1K$. [4]. Substitution of Eqs.(4)-(6) into Eq.(3) leads to the following expression:

$$P = \left[\frac{T_h}{U_H A_H (T_h - T_{hpt})(T_h - T_c)} + \frac{T_c}{U_L A_L (T_c - T_L)(T_h - T_c)} + \frac{2C_v}{R U_{reg} A_{reg} \ln(\lambda)} \right]^{-1} \quad (7)$$

For simplicity we considered the $X = \frac{T_c}{T_h}$ and

$$M = \frac{C_v}{R \ln \lambda}, \text{ so Eq. (7) can be written as follow;}$$

$$P = \left[\frac{1}{U_H A_H (T_h - T_{hpt})(1-X)} + \frac{X}{U_L A_L (X T_{hpt} - T_L)(1-X)} + \frac{2M}{U_{reg} A_{reg}} \right]^{-1} \quad (8)$$

To determine the maximum power output, take the derivative of p with respect to T_{hpt} and obtain the optimum temperature of working fluid at state 1;

$$\frac{\partial P}{\partial T_{hpt}} = 0$$

So;

$$T_{hpt} = \frac{\sqrt{U_L A_L T_L} + X \sqrt{U_H A_H T_H}}{X (\sqrt{U_L A_L} + \sqrt{U_H A_H})} \quad (9)$$

Therefore the thermal efficiency is calculated as follow;

$$\eta = \frac{1-X}{1+M(1-X)}$$

Substituting Eq. (9) into Eq. (8);

$$P_{\text{max}} = \left[\frac{1}{U_H A_H (T_h - T_{hpt})(1-X)} + \frac{X}{U_L A_L (X T_{hpt} - T_L)(1-X)} + \frac{2M}{U_{reg} A_{reg}} \right]^{-1} \quad (11)$$

For the maximum power output it should be considered $U_H A_H = U_L A_L = UA$ [4]. So Eq. (11) can be written as follow;

$$P_{\text{max}} = \left[\frac{1}{UA(T_h - T_{hpt})(1-X)} + \frac{X}{UA(X T_{hpt} - T_L)(1-X)} + \frac{2M}{U_{reg} A_{reg}} \right]^{-1} \quad (12)$$

Also the corresponding maximum net work is obtained as follow;

$$W_{\text{net,opt}} = m R T_{hpt} (1-X) \ln(\lambda) \quad (13)$$

3. Numerical Results and discussion

To have a numerical appreciation of the result, we consider the heat source temperature and cold source temperature 1300 K and 320 K respectively.

Also $U_H A_H = U_L A_L = 2 \text{ kW}\cdot\text{K}^{-1}$, $x=0.5$, $\lambda=2$

and $U_{reg} A_{reg} = 1000 \text{ kW}\cdot\text{K}^{-1}$. We consider

standard air as a working fluid with $R = 0.287 \text{ kJ}\cdot\text{K}^{-1}\cdot\text{kg}^{-1}$,

$C_v = 0.718 \text{ kJ}\cdot\text{K}^{-1}\cdot\text{kg}^{-1}$ and $m=9.3$ grams as a total mass of the working fluid.

3.1. The effects of T_H

As be shown in Fig. 3, in various UA , by increasing T_H the output power increases. Also in the certain T_H , by increasing UA , output power increases too. As is clear in Fig. 4, in various $(UA)_{reg}$, by increasing T_H the output power increases, also in certain T_H , with increasing $(UA)_{reg}$, output power is increased.

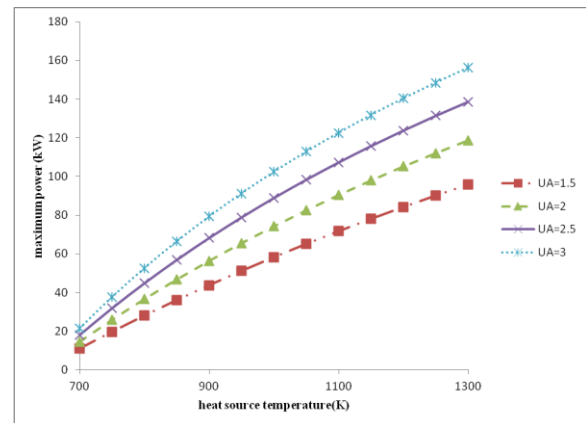


Fig.3. Variation of the power of the Stirling engine for different the heat source temperature and UA

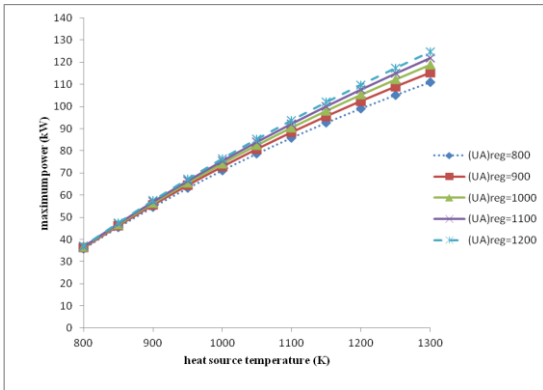


Fig.4. Variation of the power of the Stirling engine for different the heat source temperature and $(UA)_{reg}$

The effects of temperature of heat source on the output power at various values of x are presented in Fig. 5. At temperatures above 1100 K with increasing x , the power output is reduced. The slope of curve decreases by decreasing the difference between the temperature of working fluid in hot and cold spaces. According to Fig. 6, for different values of λ , with increasing T_H the output power is increased.

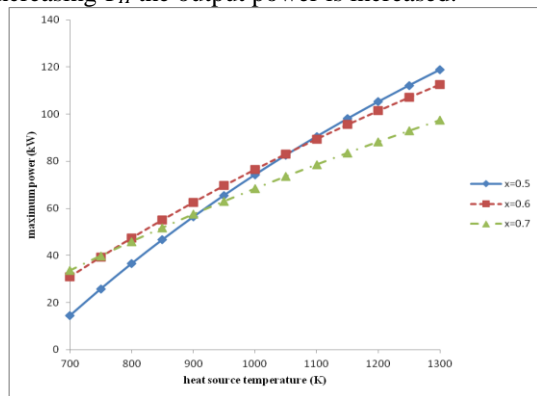


Fig.5. Variation of the power of the Stirling engine for different the heat source temperature and x

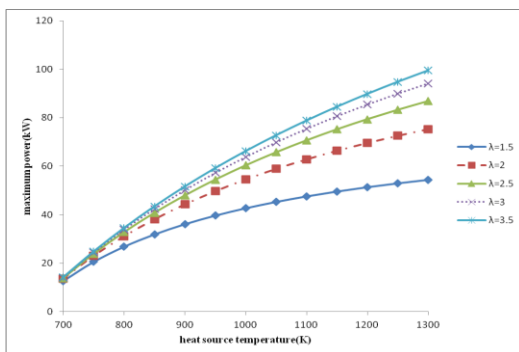


Fig.6. Variation of the power of the Stirling engine for different the heat source temperature and λ

3.2. Effects of volumetric ratio (λ)

Also in Fig. 7, the effects of volumetric ratio on the output power at various UA are presented that have similar behaviour to Fig. 6.

Fig. 8 shows the effects of the volumetric ratio on the thermal efficiency at various values of temperature ratios, in which the efficiency increases by increasing λ and as the temperature difference is decreased the thermal efficiency is reduced. Fig. 9 shows the effects of volumetric ratio on the output power for different values of x that has similar behaviour as the thermal efficiency be shown recently. Fig. 10 shows the effects of volumetric ratio on the output power for different values of $(UA)_{reg}$.

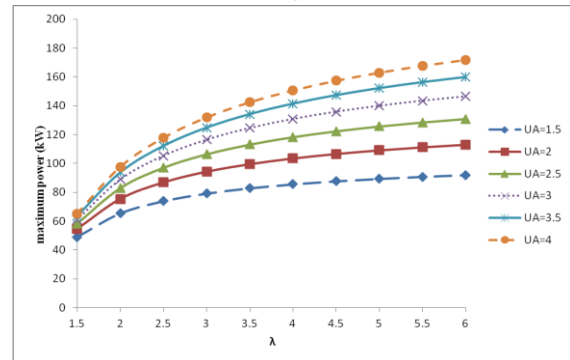


Fig.7. Variation of the power of the Stirling engine for different λ and UA

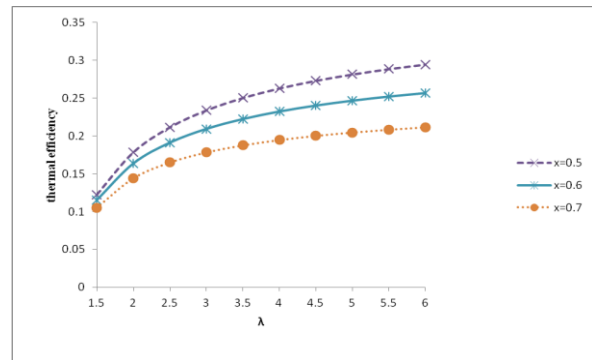


Fig.8. Variation of the thermal efficiency of the Stirling engine for different λ and x

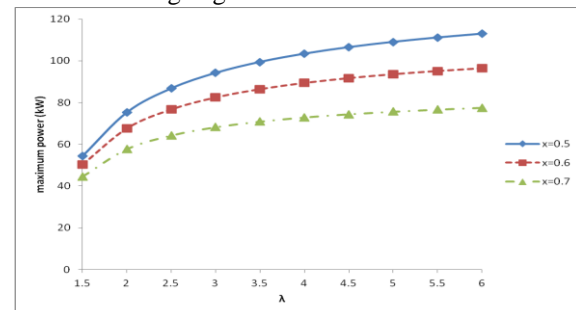


Fig.9. Variation of the maximum power of the Stirling engine for different λ and x

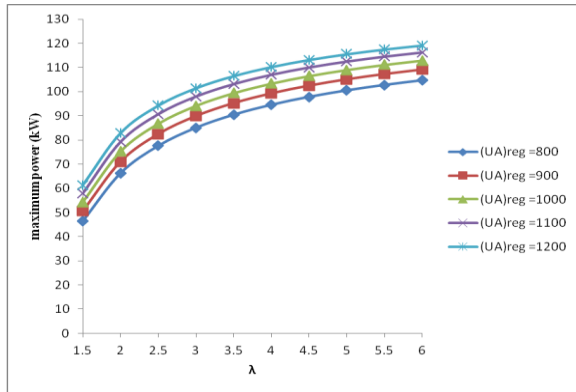


Fig.10. Variation of the maximum power of the Stirling engine for different λ and $(UA)_{reg}$

3.3. Effect of UA

By increasing the UA of heat exchangers the output power increases rapidly in comparison with volumetric ratio and heat source temperature that means the heat exchanger and regenerator with high effectiveness coefficient are more efficient, as be shown in Fig. 11.

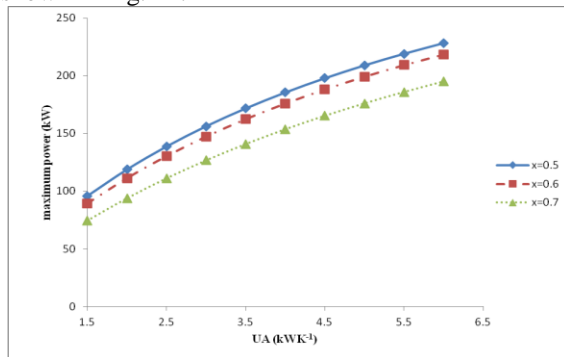


Fig.11. Variation of the power of the Stirling engine for different UA and x

Conclusions

In the presented paper, the effects of some parameters such as UA , $(UA)_{reg}$, heat source temperature (T_H) and volumetric ratio (λ) on the output power and the thermal efficiency is investigated. It can be concluded that the effects of UA is more than $(UA)_{reg}$ on the output power. The temperature of heat source is one of the important parameters affecting the output power and by increasing of temperature the power increases too. This behaviour varies with changes in other parameters.

The volumetric ratio has a effective role in output power and by increasing it, the power increases, but the slope of the curve decreases by increasing temperature ratio; Also the effects of λ in the range 1.5 to 3.5 is sensible. Volumetric ratio affects thermal efficiency similar to the output power. Also the

impacts of x on the power and thermal efficiency are evaluated. The results of this analysis show that the theory can be useful for design and the performance analysis of Stirling heat engine.

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