



STUDY ON REVIEW OF LITERATURE ON STUDY ON THE LINEAR ALGEBRIC EQUATION AND ITS APPLICATION

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Abstract: Various studies (Stacey, 1988; Vinner, 1991; Kieran, 1992; Esty, 1992; Sfard&Linchevski, 1994; Bell, 1995; Linchevski & Herscovics, 1996; McDowell, 1996; Souviney, 1996; Dreyfus, 1999; Lithner, 2000; Mason, 2000; Maharaj, 2005) have focused on the teaching and learning of school mathematics. These studies have indicated some important sources of students' difficulties in mathematics. Kieran (1992) considered a student's inability to acquire an in-depth sense of the structural aspects of algebra to be the main obstacle. Sfard and Linchevski (1994) have analysed the nature and growth of algebraic thinking from an epistemological perspective supported by historical observations. They indicated that the development of algebraic thinking was a sequence of ever more advanced transitions from operational (procedural) to structural outlooks.

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Introduction

Literature Review Numerous texts trace the history of mathematics. Many of these texts present an overview of mathematical developments. A few will select a main idea, and investigate it thoroughly. However, it is difficult to find a source that specifically traces the development of equation solving and its applications to the secondary classroom.

Algebra has been recognized as a critical milestone in students' mathematics learning. However, it has been noted that many students created a serious barrier in the algebraic problem solving and formal algebraic system (Kieran, 1992).

Math through the Ages (Berlinghoff & Gouvea, 2004) is an excellent book from which to learn the history of some key mathematical ideas. The text focuses on a few main ideas, and expands upon them. Specifically, it provides interesting stories and histories on people. However, it does not show most of the actual work that was needed to derive the formulae and ideas presented. On the other hand, *Journey through Genius* (Dunham, 1990) provides many of the proofs and derivations of formulae in addition to interesting background information. However in this book, the focus of each chapter is a specific theorem, rather than the evolution of a mathematical idea. Swetz's (1994).

In order to evaluate the related literature, 29 articles are selected from my database searching and

then categorized into a taxonomy including the five categories: algebra content, cognitive gap, teaching issues, learning matters, and transition knowledge. After that, the taxonomy is used to conduct the whole literature review. Within the taxonomy, each category is not independent. For instance, the category of algebra content is the knowledge base for the other four categories. Meanwhile, one of the articles could be coded into more than one category. For instance, the article is coded into four categories such as algebra content, cognitive gap, teaching issues, and learning matters (Kieran, 1992).

From Five Fingers to Infinity provides a broader range of topics of historical mathematics. Swetz de-emphasizes individuals, and presents the materials by geographic location and time. For instance, one chapter is specific to the evolution of mathematics in ancient China. This presentation style is quite valuable in getting such a large amount of information across. However, it lacks the interesting personal stories present in books such as *Math through the Ages* and *Journey through Genius* that can motivate the reader to investigate a topic further. *The Historical Roots of Elementary Mathematics* (Bunt, Jones, & Bedient, 1976) is very similar in style and information to *Math through the Ages*. Both books present information in short chapters specific to a main idea (e.g. Greek numeration systems). In addition, both books cover a wide range of topics that are broken down by date. However, *The Historical*

Roots of Elementary Mathematics does not delve into the stories describing the people behind the discoveries. The four volume collection *The World of Mathematics* (Newman, 1956) consists of individual articles compiled together in an effort to convey the diversity, *the utility and the beauty of mathematics* (Newman, 2002).

Newman attempted to show the richness and range of mathematics. This collection spans ideas from the Rhind Papyrus to the "Statistics of Deadly Quarrels" (Newman, 2003). *The World of Mathematics* presents an amazingly broad view of the many applications of mathematics to the sciences. *An Introduction to the History of Math* (Eves, 1956) covers the same topics as several of the other books, in much the same manner. It traces the development of mathematics from numeration systems through to the development of calculus. It includes specific information of the individuals that developed many of the critical ideas in the history of mathematics. Boyer's (1968) *A History of Mathematics* is almost entirely about Greek mathematics. It covers ancient Greek mathematics to a degree that none of the other mentioned texts do.

REVIEW OF LITERATURE

Various studies (e.g. Stacey, 1988; Vinner, 1991; Kieran, 1992; Esty, 1992; Sfard & Linchevski, 1994; Bell, 1995; Linchevski & Herscovics, 1996; McDowell, 1996; Souviney, 1996; Dreyfus, 1999; Lithner, 2000; Mason, 2000, Maharaj, 2005) have focused on the teaching and learning of school mathematics. These studies have indicated some important sources of students' difficulties in mathematics. Kieran (1992) considered a student's inability to acquire an in-depth sense of the structural aspects of algebra to be the main obstacle. Sfard and Linchevski (1994) have analysed the nature and growth of algebraic thinking from an epistemological perspective supported by historical observations. They indicated that the development of algebraic thinking was a sequence of ever more advanced transitions from operational (procedural) to structural outlooks. Mason (2000:97) has argued that "... the style and the nature of questions encountered by students strongly influences the sense that they make of the subject matter". The questions that come to the mind of an educator are influenced by the perspective and disposition that he/she has towards mathematics and pedagogy (Mason, 2000).

Perhaps one of the most valuable tools for a secondary teacher available is *Historical Topics for the Mathematics Classroom* (National Council for Teachers of Mathematics, 1989). This text consists of a series of "capsules" (short chapters). Each capsule

gives a brief historical overview of a particular topic (e.g. Napier's Rods). The capsules are grouped by general topic (algebra, geometry, trigonometry, etc.). Specifically, this text provides a historical context to graphical approaches to equationsolving.

Filloy and Rojano (1989) defined one of the fundamental ruptures between arithmetic and algebra is a didacticalcut. The notion referred to the transition that occurred as students face such equations as

$$ax + b = cx + d$$

. Students could successfully solve the equation as $ax + b = c$ using reversal operation as subtracting B from D and dividing by A. This type of equation was called by them as "arithmetical" (p. 19). The reversal operation is not applicable for the non-arithmetical equations as

$$ax + b = cx + d$$

. In order to solve such equations, students have to resort to a truly algebraic idea of operating the unknown (Radford, 2012). Operating the unknown requires students to think analytically, treating the unknown as if it is known (Radford & Puig, 2007). This view provides a specific situation which requires the transition from arithmetic to algebra. Certainly, such requirement stems from the structural nature of algebra.

In addition, it provides a concise overview of the methods employed to solve quadratics and cubics. The people that developed these methods are named, though little is said about their personal history. The many texts available on the history of mathematics all attempt to convey an enormous amount of information in different ways. Some briefly describe many of the contributions that people have made to mathematics. Others describe the contributions of a culture, paying less attention to individuals (thereby allowing more time for the derivations of formulae). Although many texts include the evolution of equation solving in their exposition, such material is often spread throughout the text. In addition, most texts are not geared specifically for secondary teachers. I believe my project will complement this body of knowledge. As opposed to covering the breadth of mathematics, I will focus on equation solving in a way that will connect to the secondary classroom. My intention is to provide a source that can help secondary teachers understand where their textbook formulae came from and to familiarize teachers with some of the people and stories that contributed to the development of the mathematics we use today. I would like to demonstrate that the problem solving techniques for equations that are used in the secondary high school curriculum today did not simply fall from the sky! Rather, modern methods for solving equations took time, dedication and effort to evolve.

Kieran (1992) had offered a historical account of the development of algebraic symbolism and its transformational rules, which emphasized the distinguished features of letters between representing unknowns in equation solving and representing givens in expressing general solutions. Furthermore, Kieran (1992) analyzed that the development of algebraic symbolism demonstrated a change from a procedural to a structural perspective on algebra. Meanwhile, the structural development of algebra has a considerable impact on school algebra learning.

In school mathematics, arithmetic is normally treated as numerical computations (Sfard & Linchevski, 1994). Arithmetic method is used to carry out one or more operations with given numbers to achieve a solution. For elementary algebra, its need is to define the relationships between the unknown and the known data in a problem. As Sadosky and Sessa (2005: p. 90) pointed out, "the 'object' of arithmetic in primary school is numbers, whereas elementary algebra focuses on relationships between quantities". It is also shown that students' prior exposure in computing binary operations does not prepare them very well to handle algebra (Banerjee & Subramaniam, 2012). For instance, students often apply procedures that have been employed in arithmetic context to simplify algebraic expressions and make the similar mistakes (Fischbein & Barash, 1993).

The cognitive load work by Kirschner, Sweller, and Clark (2006) gives an explanation for the necessity of fluency with prerequisite knowledge. Without prerequisite fluency, short-term memory becomes overloaded and unable to effectively process the new concepts being learned.

Boyer's (1968) *A History of Mathematics* is almost entirely about Greek mathematics. It covers ancient Greek mathematics to a degree that none of the other mentioned texts do. Perhaps one of the most valuable tools for a secondary teacher available is *Historical Topics for the Mathematics Classroom* (National Council for Teachers of Mathematics, 1989).

This text consists of a series of "capsules" (short chapters). Each capsule gives a brief historical overview of a particular topic (e.g. Napier's Rods). The capsules are grouped by general topic (algebra, geometry, trigonometry, etc.). Specifically, this text provides a historical context to graphical approaches to equationsolving. In addition, it provides a concise overview of the methods employed to solve quadratics and cubics.

Various researchers (Vaiyavutjamai & Clements, 2006) have illustrated that very little attention has been paid to quadratic equations in mathematics education literature, and there is scarce

research regarding the teaching and learning of quadratic equations.

A limited number of research studies focusing on quadratic equations have documented the techniques students engage in while solving quadratic equations (Bossé & Nandakumar, 2005), geometric approaches used by students for solving quadratic equations (Allaire & Bradley, 2001), students' understanding of and difficulties with solving quadratic equations (Kotsopoulos, 2007; Lima, 2008; Tall, Lima, & Healy, 2014; Vaiyavutjamai, Ellerton, & Clements, 2005; Zakaria & Maat, 2010), the teaching and learning of quadratic equations in classrooms (Olteanu & Holmqvist, 2012; Vaiyavutjamai & Clements, 2006), comparing how quadratic equations are handled in mathematics textbooks in different countries (Saglam & Alacaci, 2012), and the application of the history of quadratic equations in teacher preparation programs to highlight prospective teachers' knowledge (Clark, 2012).

In general, for most students, quadratic equations create challenges in various ways such as difficulties in algebraic procedures, (particularly in factoring quadratic equations), and an inability to apply meaning to the quadratics. Kotsopoulos (2007) suggests that recalling main multiplication facts directly influences a student's ability while engaged in factoring quadratics. Furthermore, since solving the quadratic equations by factorization requires students to find factors rapidly, factoring simple quadratics becomes quite a challenge, while non-simple ones (i.e., $ax^2 + bx + c$ where $a \neq 1$) become harder still. Factoring quadratics can be considerably complicated when the leading coefficient or the constant term has many pairs of factors (Bossé & Nandakumar, 2005).

The research of Filloy & Rojano (1989) suggested that an equation such as with an expression on the left and a number on the right is much easier to solve symbolically than an equation such as $ax + b = c$. This is because the first can be 'undone' arithmetically by reversing the operation 'multiply by 3 and subtract 1 to get 5' by 'adding 1 to 5 to get 6 and then dividing 6 by 3 to get the solution.

Meanwhile the equation cannot be solved by arithmetic undoing and requires algebraic operations to be performed to simplify the equation to give a solution. This phenomenon is called 'the didactic cut'. It relates to the observation that many students see the 'equals' sign as an operation, arising out of experience in arithmetic where an equation of the form $a + b = c$ is seen as a dynamic operation to perform the calculation, 'three plus four makes 7', so that an equation such as $ax + b = c$ is seen as an operation which may possibly be solved by arithmetic 'undoing' rather than requiring algebraic manipulation (Kieran, 1981).

References:

- [1]. Gilbert, S. (2009). Introduction To Linear Algebra (4th ed.). United Kingdom: Wellesley-Cambridge Press.
- [2]. Katta, G. (2014). Computational and Algorithmic Linear Algebra and n-Dimensional Geometry. USA: World Scientific Publishing.
- [3]. Bretscher, O. (2004). Linear Algebra with Applications (3rd ed.). New York, NY: Prentice Hall.
- [4]. Jolliffe, I.T. (1986). Principal Component Analysis. New York, NY: Springer-Verlag.
- [5]. Mashal, N., Faust, M., & Hendler, T. (2005). The Role Of The Right Hemisphere In Processing Nonsalient Metaphorical Meanings: Application Of Principal Components Analysis To fMRI Data. *Neuropsychologia*, 43(14), 284-300.
- [6]. Gonzalez, R.C., & Woods, R.E. (1992). Digital Image Processing. Massachusetts: Addison- Wesley.

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