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Implementing the Ensemble Kalman Filter Method as a Novel Tools for Interpretation of the Pressure Transient Tests

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Abstract: Nowadays conventional well test analyses, such as straight line or type curve are used to estimate reservoir parameters. There are some challenges and problems in their procedure. Conventional well test methods do not provide such data. In addition, analyzing of transient pressure well test data contaminated with disturbance and noise with conventional methods is often difficult. Recent reservoir management methods and uncertainty analyses require probability distribution function of reservoir properties in each time step. Moreover the test duration maybe short or missing data and some assumptions are used to derive the mathematical formulations. For this reasons, we move on a new and more robust formulation of well test data analysis to eliminate all these problems. The ensemble Kalman filter (EnKF) is a sequential data assimilation base on Monte Carlo approach, in which an ensemble of models, instead of only one model as in traditional history matching methods and other Kalman filter related methods, is used. In this paper we use Ensemble Kalman Filter methods, for the first time, to interpret oil well test data and estimate unknown parameters in an on-line approach. In this method we convert the well test problem in to state space framework to use system identification procedures (in time-domain) to find solution of our stochastic problem. The Kalman filter procedure for integration consists of two steps: a forecast step and an update step. Mathematical formulations and measurement data are used in the forecast step and updating respectively. A synthetic example is used to examine the validity and effectiveness of the method. We made use of a real well test data with a lot of noise and missed data as well. For any example, first pure wellbore storage region and middle time region are identified then after receiving any measurement data, both dynamic variables, such as pressure and static variables as permeability, skin factor, wellbore storage coefficient and distance from fault with their probability density function (PDF) and standard deviation are updated. The results show that the Ensemble Kalman Filter method can be used as a powerful tools for well test analysis.

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1. Introduction

The Kalman Filter (Kalman, 1960) is the most common filtering technique for linear Gaussian models. The Kalman filter has historically been the most widely applied method for assimilating new measurements to continuously update the estimate of state variables.

Kalman filters have occasionally been applied to the problem of estimating values of petroleum model variables (Eisenmann et al., 1994; Corser et al., 2000), but they are most appropriate when the problems are characterized by relatively small numbers of variables and when the variables to be estimated are linearly related to the observations.

In essence, the Kalman filter state estimate is a weighted linear combination of the background (forecast) state and observations, where the weights depend on the uncertainties in model predictions and observations. For smaller observation errors (relative to the prediction errors) the analysis states are drawn closer toward the observation whereas for very uncertain observations model predictions are weighted more.

Most data assimilation problems in petroleum reservoir engineering are highly non-linear and are characterized by many variables, often two or more variables. Application to non-linear problems was at least partially solved by the development of the extended Kalman filter. However, it did not solve the critical problem with non-linear unstable dynamics, where it leads to a linear instability in the error covariance evolution (Evensen, 1994). The problem of weather forecasting is in many respects similar to the problem of predicting future petroleum reservoir performance. The economic impact of inaccurate predictions is substantial in both cases, as is the difficulty of assimilating very large data sets and updating very large numerical models. And it does not provide a practical solution for highly nonlinear problems. Ensemble type filters were developed to remove linear error propagation constraint and avoid the computational cost associated with covariance propagation. Evensen (Evensen,1994) proposed an ensemble version of the Kalman filter, the Ensemble Kalman Filter (EnKF), which can be used with any nonlinear state-space model.

The EnKF has gained popularity in several large scale applications such as oceanography, metrology, and hydrology (Houtekamer ,et al.1998 ; Reichle ,et al.2002). However, its application to reservoir history matching has only recently been considered. In the first use of EnKF in reservoir modeling, Ncevdal et al. (Navdal ,et al. 2002) updated near-well reservoir models by adjusting the permeability field. They found that early measurements were more important in tuning the permeability field than the later ones. Lorentzen et al. (Lorentzen , et al. 2003) successfully used EnKF with a two-phase flow model to tune model parameters in underbalanced drilling. The ensemble Kalman filter is an attractive option for properties of reservoir estimation in real-time reservoir control applications. It is easy to implement, provides considerable flexibility for describing geological heterogeneity, and supplies valuable information about prediction uncertainty.

1. Ensemble Kalman filter

Kalman filter based methods perform sequentially and only update the model with the latest available data. An assimilation step is implemented to modify the model parameters, based on the difference between reservoir simulation responses and the data from the field. The updated model is then used to run forward until reaching the next measurement time.

Different from the general Kalman filter, the EnKF runs multiple simulation models independently, assimilates only the new measurements, and updates the multiple models simultaneously. After each updating, the EnKF describes model parameters through two statistical properties: mean and variance, the first representing the most probable model and the second depicting the change range, i.e., uncertainty. Aside from the initial sampling, the EnKF consists of two steps for each time-recursive process: a forecasting step based on current state variables (which solves the flow equations with current static and dynamic parameters) and an assimilation step (which updates the state variables).

The forecast step for the EnKF propagates the state vectors forward in time from a previous measurement time, using the estimates of the variables conditional to all the observed data up that time, to current measurement time.

$$x_k^p = \psi(x_{k-1}^u) + w_{k-1} \quad 1$$

Where ψ represents the reservoir flow equations. It relates the state at the previous time step $k-1$ to the state at the current time step k . It includes the zero-mean process noise w_k and covariance Q_k, i.e. $E[w_k {w_k}^T] = Q_k$.

We define ${x^p}_k$ (note the "p") to be our a priori state estimate at step k given knowledge of the process prior to step k, and ${x^u}_{k-1}$ to be our a posteriori state estimate at step *k-1* given measurement y_{k-1}.The relationship between the perturbed observation and the true state vector can be written as:

$$y_k = Hx_k + v_k \quad 2$$

where v_k is the perturbation added to the noisy measured data; v_k is Gaussian distributions with mean 0 and covariance $R_{,k}$, i.e. $E[v_k {v_k}^T]=R_{,k}$; the process noise w_k and perturbation noise v_k are uncorrelated in time (white), i.e. $E[w_k {v_k}^T] =0$ for all *k*. In practice, the process noise covariance Q_k and measurement noise covariance R_k matrices might change with each time step or measurement, however here we assume they are constant.

Where H is an operator matrix or row vector, depending on the number of observations, which relates the state vector to theoretical data. H is a trivial matrix whose elements are only ones or zeroes. We can always arrange it as:

$$H = [0 \mid I] \quad 3$$

The mean vector of the variables in the state vectors is computed from the following equation:

$$\overline{x_k^p} = \sum_{i=1}^{Ne} x_k^p i \quad 4$$

The matrix L is defined as:

$$L_k^p = \left[x_k^p 1 - \overline{x_k^p} \quad x_k^p 2 - \overline{x_k^p} \quad \ldots \quad x_k^p Ne - \overline{x_k^p}\right] \quad 5$$

The matrix P_k^p is an approximation to the model error covariance matrix, that is estimated from the ensemble at any time using the standard statistical formula:

$$P_k^p = \frac{1}{Ne-1} L_k^p {L_k^p}^T \quad 6$$

The weighting matrix is the Kalman gain matrix and denoted by K_k.

$$K_k = P_k^p H^T (H P_k^p H^T + R_k)^{-1} \quad 7$$

In deriving the equations for the updating step in EnKF, we begin with the goal of finding an equation that computes an a posteriori state estimate x_k^u as a combination of a priori x_k^p estimate and a weighted difference between an actual measurement y_k and a measurement prediction Hx_k^p as shown below in equation (8).

$$x_k^u = x_k^p + K_k(y_k - Hx_k^p) \quad 8$$

Here, y_k is perturbed observation. It means that a random error with mean zero and a covariance matrix of measurement error (R_k) is added to real observation to avoid Kalman gain collapse due to sampling error.

The difference $(y_k - Hx_k^p)$ in equation (8) is called the measurement innovation, or the residual. The residual reflects the discrepancy between the predicted measurement Hx_k^p and the actual measurement y_k. A residual of zero means that the two are in complete agreement.

Finally, The updated error covariance matrix P_k^u, of the model can be computed along the same lines as P_k^p, the model error covariance matrix after the analysis step is:

$$P_k^u = (I - K_k H) P_k^p \quad 9$$

The above is an illustration of the two-step procedure at one measurement time.

1. General mathematical formulation in well testing

In this section, a brief overview of the governing equations in reservoir simulation and well testing are presented (Bourdet , 2002 ; Horne,1995). These equations will be referred as dynamic forward models in describing the data assimilation algorithm. The equation governing the flow is normally written in terms of the pressure. Since we assume radial symmetry, the pressure P depends only on the radius r and time t, the equation is as following:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right) = \frac{1}{\eta}\frac{\partial P}{\partial t} \quad 10$$

Here, η is diffusivity constant and is equal to $\eta = 0.0002637k/\varphi\mu c$ and r (ft), P (psi), φ (fraction), μ (cp), c (psi^{-1}), k (md), t (hr) are the radius from well center, reservoir pressure, porosity, total compressibility, permeability and time, respectively.

This is the so-called diffusivity equation and it is considered one of the most important mathematical expressions in petroleum engineering. This equation is derived under the assumption that the permeability and viscosity are constant over pressure, time and distance ranges. The fluid is assumed to be slightly compressible, like, say, oil. The notation $\frac{\partial P}{\partial r}$ and $\frac{\partial P}{\partial t}$ means partial derivative with respect to r and t respectively.

For infinite cylindrical reservoir we assume that (1) a well produces at a constant flow rate q; (2) the well has zero radius; (3) the reservoir is at uniform pressure P_i, before production begins; and (4) the well drains an infinite area (i.e. that $P \to Pi$ as $,r \to \infty$). The solution of the diffusivity equation is given by the expression:

$$P_i - P_{wf} = -70.6\frac{qB\mu}{kh}\left[Ei\left(-\frac{948\varphi\mu c_t r_w^2}{kt}\right) - 2s\right] \quad 11$$

Here, P_i *and* P_{wf} are the in initial pressure and well flowing pressure in psi, q is flow rate of oil (stb/day), B is oil formation volume factor (bbl/stb), s is skin factor and h is reservoir thickness.

The E_i function or exponential integral is:

$$Ei(-x) = -\int_x^\infty \frac{e^{-u}}{u} du \quad 3$$

The most important parameter in early time is wellbore storage coefficient (c_s), when the wellbore is shut in at the surface, the fluid will continue to flow into or out of the wellbore until pressure is equalized between the wellbore and formation. It is calculated from the early time data we use equation (12) to estimate wellbore storage coefficient.

$$c_s = \frac{qB}{24}\frac{\Delta t}{\Delta P} \quad 12$$

The important case in many reservoirs at latter time is appearance of a fault. Superposition helps us to derive a general equation that is a function of time and distance from well to fault. The equation describing a reservoir with fault is:

$$P_i - P_{wf} = -70.6\frac{qB\mu}{kh}\left[\ln\frac{1688\varphi\mu c_t r_w{}^2}{kt} + Ei\left(-\frac{948\varphi\mu c_t(2L)^2}{kt}\right) - 2s\right] \quad 13$$

From the above equation, L (distance from well to the fault) is estimated.

1.1. Convert the mathematical formulation to the state space model

To solve the problem here with EnKF theory, we should at first convert the well test formulations into a state-space model. State space model is a representation framework for dynamic of system. To do so, it is required that dynamic process model and measurements model be formulated for given problem.

We define state of system $x_t = [P_{wf}]$ be well flowing pressure and system parameter vector $\theta = [parameters]^T$. Here, T represents transpose of vector system dynamic model in state space form can be displayed as (Lewis ,et al. 2006) :

$$x_t = F_t(x_{t-1}, \theta) + w_t \quad 14$$

F_t shows nonlinear relationship between well flowing pressure and unknown parameters and is a function of time. We will assume that due to some assumption made during derivation of equations, there is certainly some error associated with model dynamics. So we show that error with w_t which is a additive Gaussian noise with mean zero and variance $\sigma_w{}^2$ described with a normal probability distribution of $N(0, \sigma_w{}^2)$. We suppose that through time, our observation, y_t, will be bottomhole pressure and it will be measured with some interval period. Now, we are able to present the measurement model for state space representation as:

$$y_t = x_t + v_t \quad 15$$

As, it is clear, any measurement is noisy and it is included here through v_t which is a additive Gaussian noise with mean zero and variance $\sigma_v{}^2$ distributed with a normal probability distribution of $N(0, \sigma_v{}^2)$ equations (14) and (15) define our state space model, cooperatively.

These equations are the core for EnKF implementation. The aim, here, is to utilize the concept of state space modeling to estimate parameter vector of the system. To do so, we use state space augmentation procedure known as self-organizing state space model to compute state and parameters of system simultaneously. At first we should modify state vector of system as $x_t = [P_{wf}, \theta]^T = [P_{wf}, k, s]^T$ to embed parameter evolution through time. It should be noted that parameters are constant and not vary with time but we impose time dependency into them to provide their variability through time and during the identification period. Thus, following equations are added to system dynamic model:

$$k_t = k_{t-1} + w_k \quad 16$$

$$s_t = s_{t-1} + w_s \quad 17$$

$$c_t = c_{t-1} + w_c \quad 18$$

$$L_t = L_{t-1} + w_L \quad 19$$

Where w_s ,w_k , w_c and w_L are some noise with decreasing variance over time to model the uncertainties therein .

2. The general algorithm

The procedure in the following is intended to provide a general idea of the basic computations and the reasons for computational efficiency of the EnKF algorithm. The general algorithm is summarized below:

Initialization Step:

1. Sets of N realizations from prior knowledge for any unknown parameters are generated.

Forward Step:

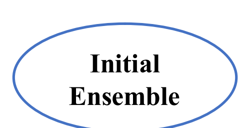
2. Each realization is replaced in the dynamic model equations (10 to 13) for calculations of the predicted realizations $d^p{}_k$, the superscript “p” denotes the prior.

$$d^p{}_k = [p1\ \ p2\ p3\ p4\ p5\ , \dots , pNe]_k^p \quad 20$$

Assimilation Step:

3. The mean of two previous steps are calculated from equation (4).
4. The matrixes L and P_k^p are computed from equations (5, 6), respectively.
5. The Kalman gain matrix is computed from equations (7).
6. By arriving the new observation y_k, each realization and state error covariance matrix are updated from equations (8, 9), respectively.
7. If there are additional data, return to Step 2 with initial guess from previous step.

The interval between two consecutive measurement times might be as short as a few seconds, such as data from permanent gauges, or as long as hours, days. The two-step procedure is repeated at each measurement



time till the last measurements are assimilated. Figure 1 illustrates the basic workflow chart of the EnKF.

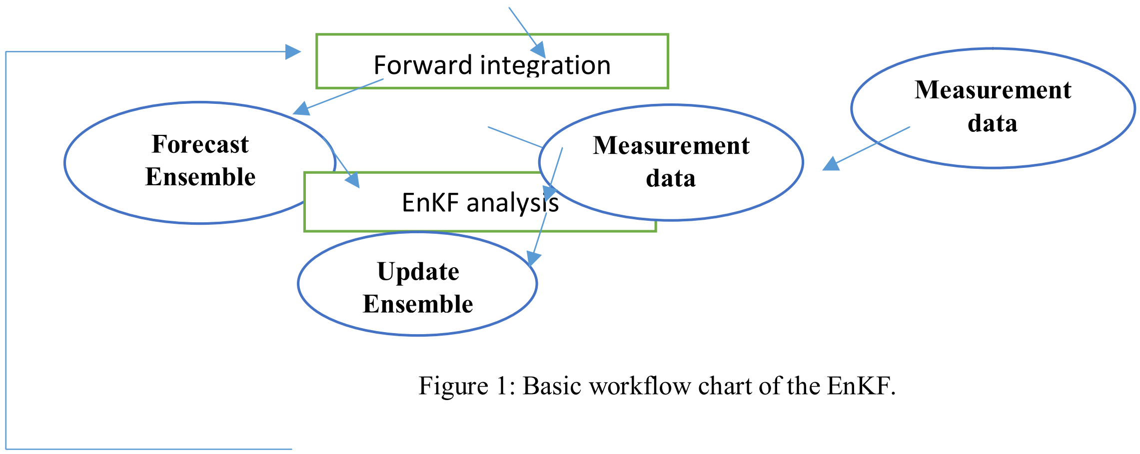


Figure 1: Basic workflow chart of the EnKF.

3. Numerical Implementation of EnKF

5.1 Synthetic well test data

This section is used for validation of the method. In the examples we present, all measurements are generated synthetically by running the model with a given numerical values in Table 1, and adding noise to the obtained values to generate measurements.

This method calculates parameters step by step. If the next data is very different from the previous data or the noise of data is increased by the time, these properties will not converge and deviate from true value. From this point of view, we can guess the time that the pressure response reaches to the external boundary or the regime changes or there may be some problems about observational.

In this research, first we should identify the regions of earlier time, middle time and late time for estimation of wellbore storage coefficient, skin factor, permeability and distance from well to the fault. We estimated these properties from this synthetic draw down well test data and then compared with their true values. Of course, some noise are added to synthetic well test data to approach to the real situation.

Table 1: Parameters used in the equations for synthetic oil well test data generation.

Parameters	Value	Parameters	Value
Initial Pressure psi	5000	Reservoir Thickness (ft)	30
Oil Rate (STB/D)	400	Formation Volume Factor (B_o) RB/STB	1.25
porosity	0.2	Wellbore Radius (ft)	0.24
Compressibility (psi-1)	12e-6	Oil Viscosity (cp)	1.5
Skin factor	8	Permeability (md)	80
Wellbore storage coefficient (bbl/psi)	0.04	Distance from well to the fault (ft)	1210

Calculation of the wellbore storage coefficient needs some of data which are lied on straight line with unit slope in log-log plot. In particular, they should be after opening of well.

Wellbore storage is normally assumed to be constant during a test and, in practice, this assumption is often reasonable. However, there are numerous situations where wellbore storage is not constant, passing of earlier time to middle time is one of them. From this point of view, the end of pure wellbore storage can be determined. In Figure 2, 100 measurement data were assimilated. This figure shows that the ensemble mean wellbore storage is approaching to a constant value (true value 0.04 bbl/psi) and the model perfectly matches observation data from step 1 to 40. After 40th time step model and observation data deviate from each other highly and mean wellbore storage increase with time. It can arguably be concluded from the trend that end of pure wellbore storage is time step 40.

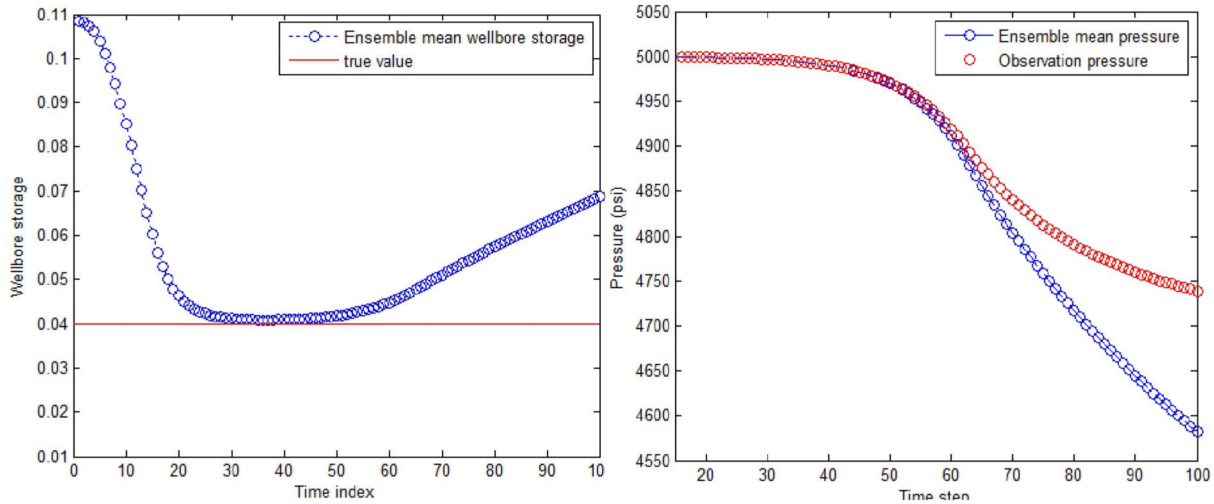


Figure 2: Ensemble mean wellbore storage through time (left) and ensemble mean pressure with observation pressure through time (right).

End of middle time is determined by the same way. Permeability and skin factor are estimated using middle time region. After middle time we expect to see the effect of boundary. Figure 3 shows if 120 of data are used to estimate the permeability, from step 1 to 60, ensemble mean permeability is trying to approach the constant values (80md) and model and observation data are matching with each other. After time step 60 that fault effect has been observed, model and observation data deviate from each other highly and ensemble mean permeability decrease with time, from this trend we can understand that end of middle time is time step 60.

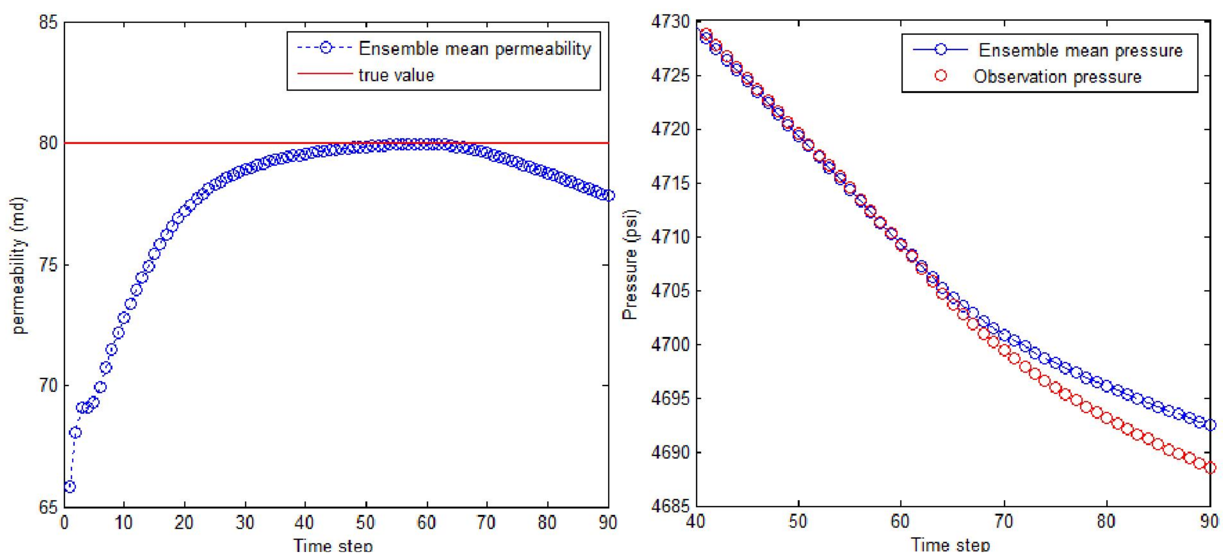


Figure 3: Ensemble mean permeability through time (left) and ensemble mean pressure with observation pressure through time (right).

Now, the parameters of its region time can be calculated. Figure 4 shows 1000 initial realizations with uniform distribution are used for filter implementation. More mathematically, it can be said that probability of wellbore storage, permeability, skin factor and distance from well to the fault shown in figure 4 is nearly for all data the above-mentioned range. This initial guesses tend towards closure when a new measurement arrives.

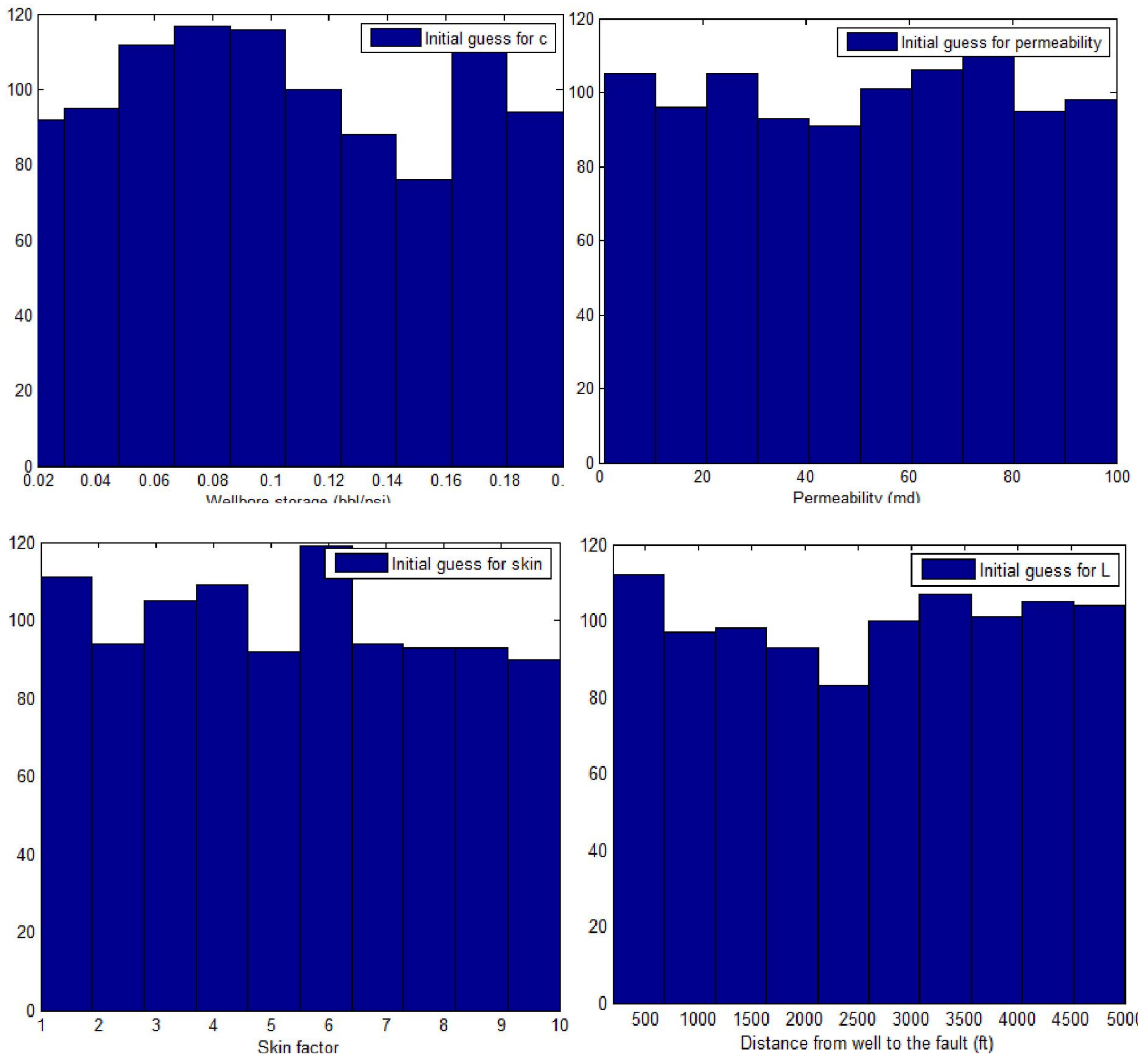


Figure 4: Histogram of initial realization without data assimilation.

Figure 5 shows probability density function of each parameter, after assimilation of second measurement. Comparing figure 5 and 4, it can be said that that EnKF is trying to modify our initial belief into a new probability density function (PDF), Being closer than the previous one.

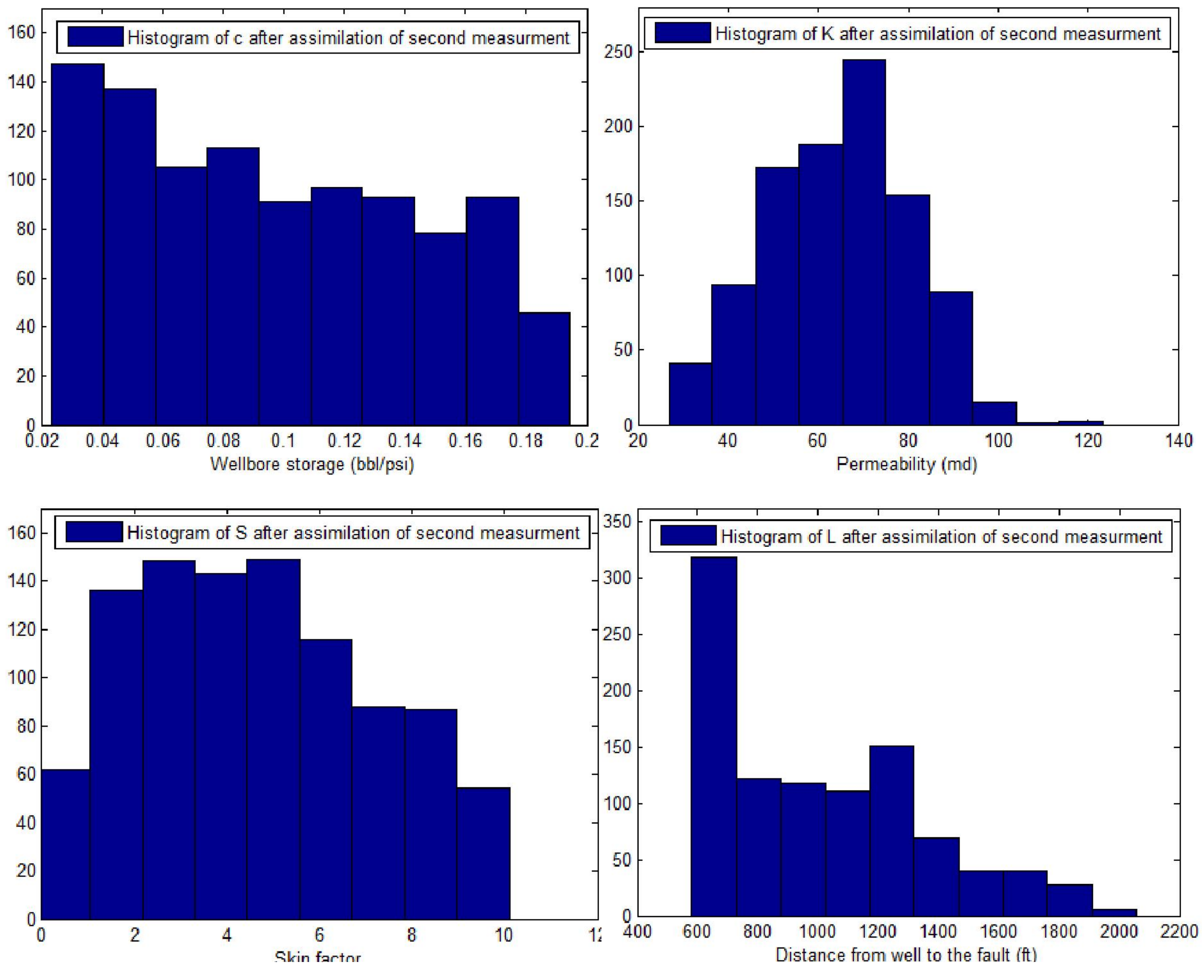


Figure 5: Histogram of realizations after assimilation of second measurement.

Figure 6 shows the histograms of each parameter after assimilation of the last measurement data. Comparison of figure 6 and 4 shows that range of data for wellbore storage coefficient, skin factor, permeability and fault are reduced from [0-0.2] to [0.0395-0.043] and [0-10] to [7.85-8.2] and [0-100] to [79.2-81.5] and [200-5000] to [1210-1213] respectively. Table 2 represent the summarized of final results from synthetic well test data using ensemble Kaman filter with their details.

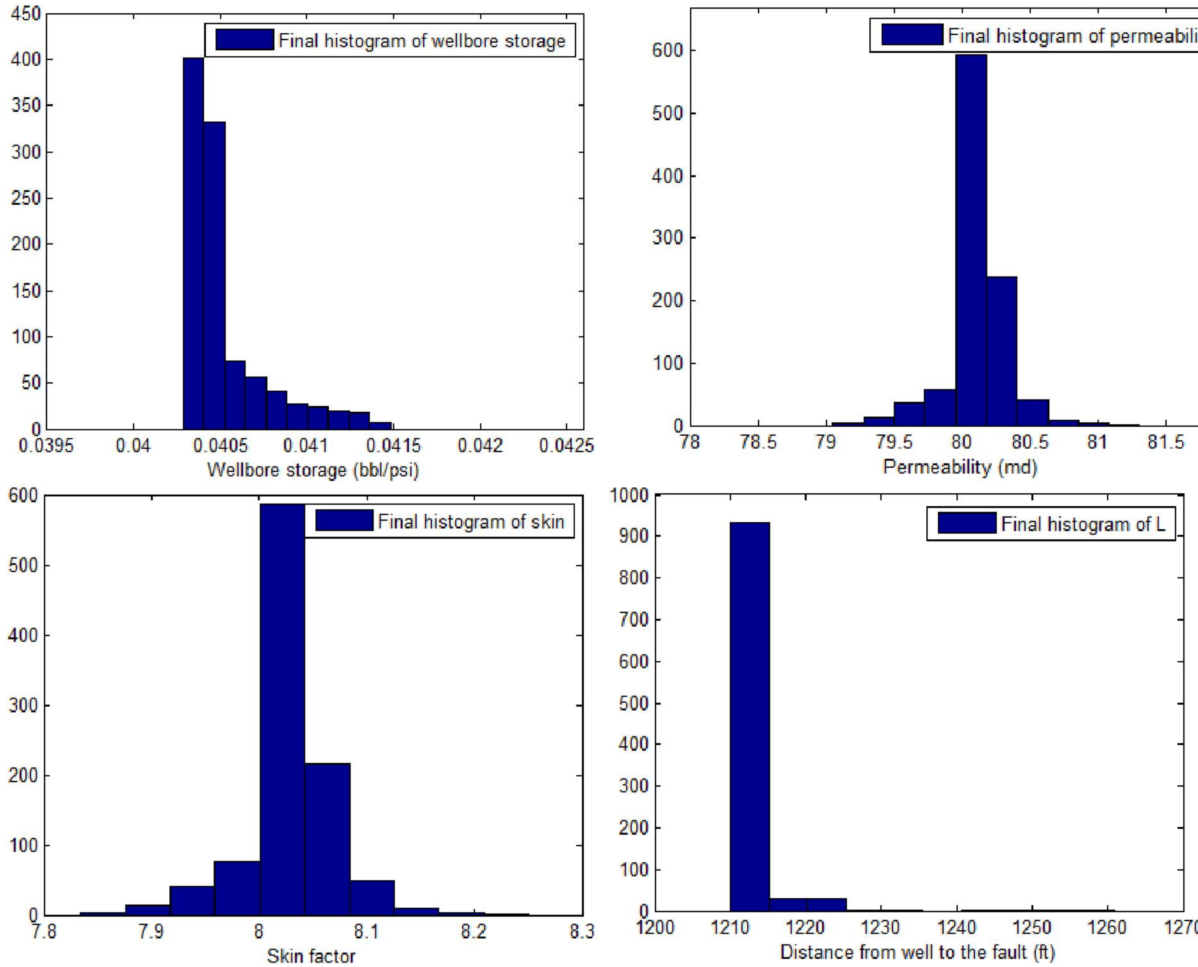


Figure 6: Histogram of final assimilation measurement.

Table 2: Summary of results from synthetic well test data using ensemble Kaman filter.

Unknown Parameter	**True value**	**Mean final**	**range**	R_k	**Final STD**
Wellbore storage coefficient (bbl/psi)	0.04	0.040832	[0.0395-0.043]	0.1	0.00064478
Permeability (md)	80	80.1159	[79.2-81.5]	4	0.21836
Skin factor	8	8.0266	[7.85-8.2]	4	0.042079
Distance from well to fault (ft)	1210	1210.9972	[1210-1213]	1	1.1709

istory of ensemble mean parameters through time are shown in Figure 7. From this figure the determination of unknown parameters with EnKF is feasible and the final values are very close to true values used in synthetic data generation. True values of unknown parameters can be obtained from mean of ensemble.

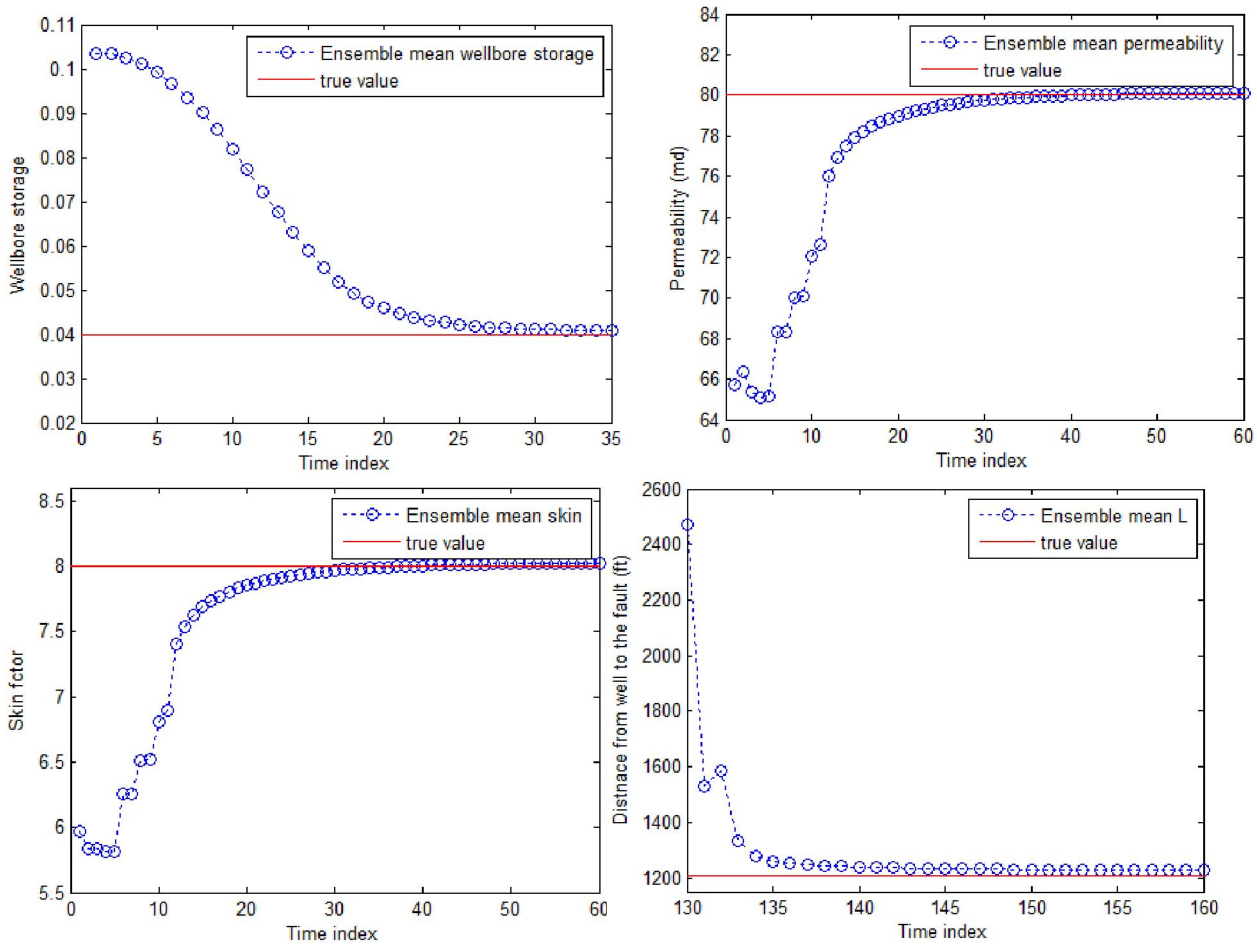


Figure 7: History of ensemble mean parameters through time.

5.2 Real Well test Data

In this section to see the effectiveness of EnKF, we use a real drawdown well test data with much noise and several time periods of this test are missed. Figure 8 shows the plot of this real well test data. The test lasted for 600 hrs. We do not have any data between 56-100 hr and 200-300 hr. In the implementation of this test with EnKF, Figure 4 was used to make the initial guesses.

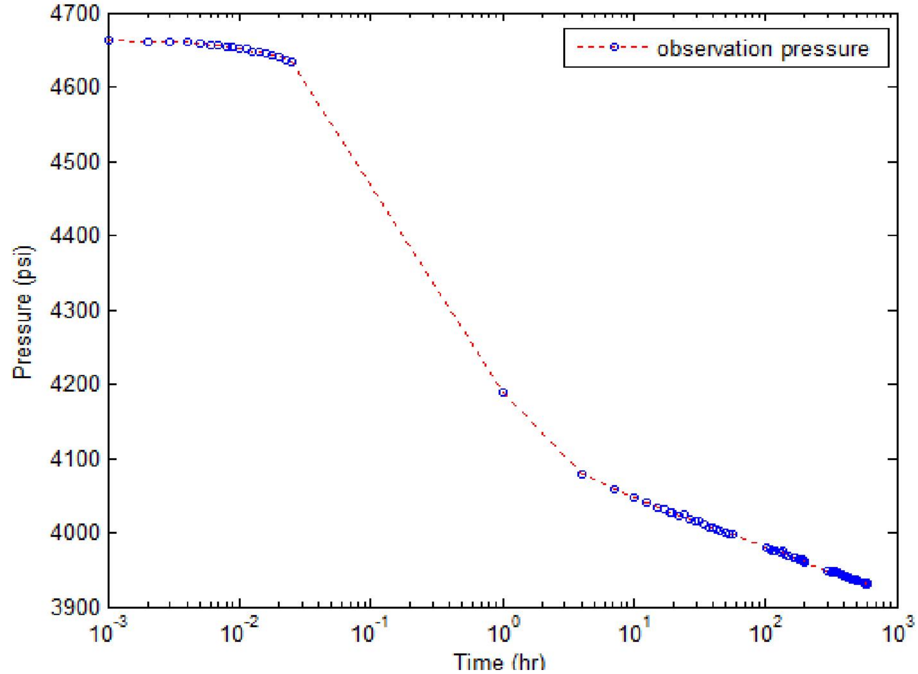


Figure 8: Plot of bottom-hole pressure versus time for a real well test.

Figure 9 shows that only 12 points are in earlier time and after passing 5 time steps wellbore storage tries to close to a true value which is 0.023 (bbl/psi). After time step 12 wellbore storage deviate from true value and increase with time step.

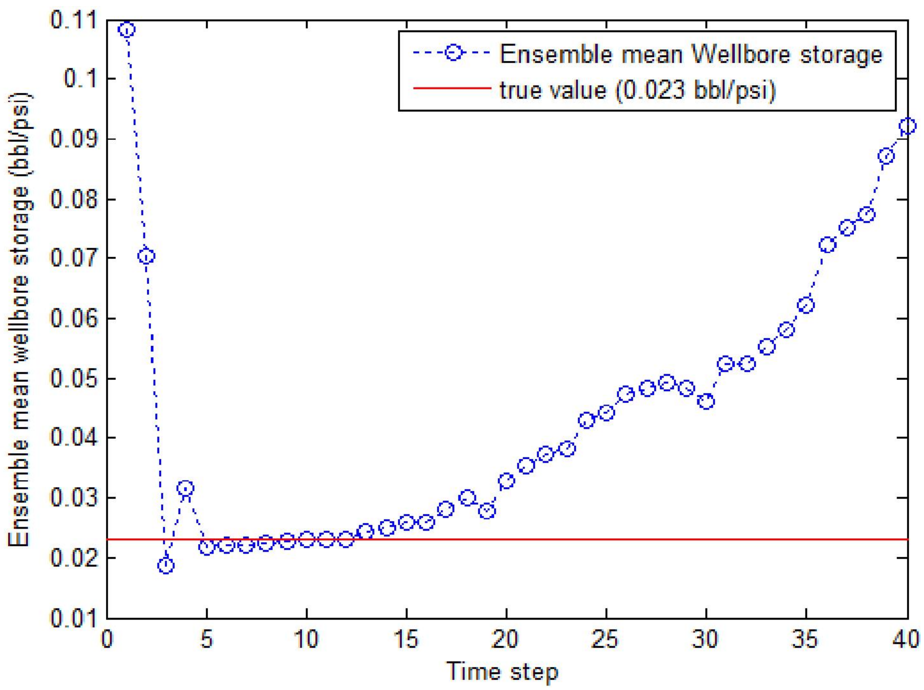


Figure 9: Ensemble mean of wellbore storage through time for real well test.

Figure 10 shows the ensemble mean unknown parameters through time. This figure shows at early observed data filter cannot obtain much information from measurement because initial ensemble mean is far from the real value and the well test data have more noise. After passing some time steps ensemble mean converges to the true value. Trend of this figure for permeability and skin is similar.

Figure 11 shows the histograms of each parameter after assimilation of the last measurement data. This figure shows the probability distribution of skin and permeability are almost similar.in addition, Figure 10 shows that their trends are similar. Details of results after final assimilation of measurement are shown in table 3.

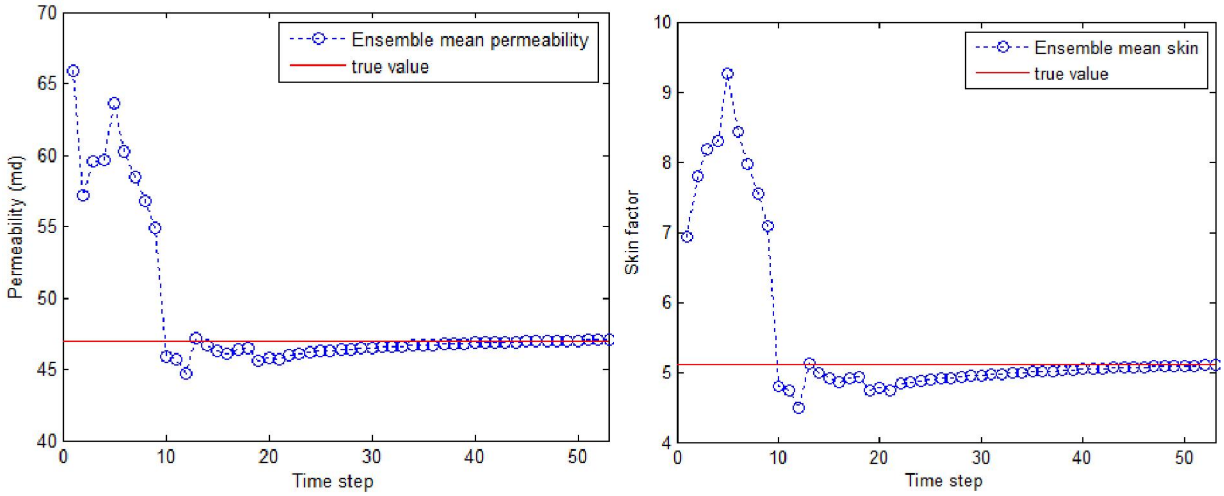


Figure 10: Ensemble mean of permeability (left) and skin factor (right) through time for real well test.

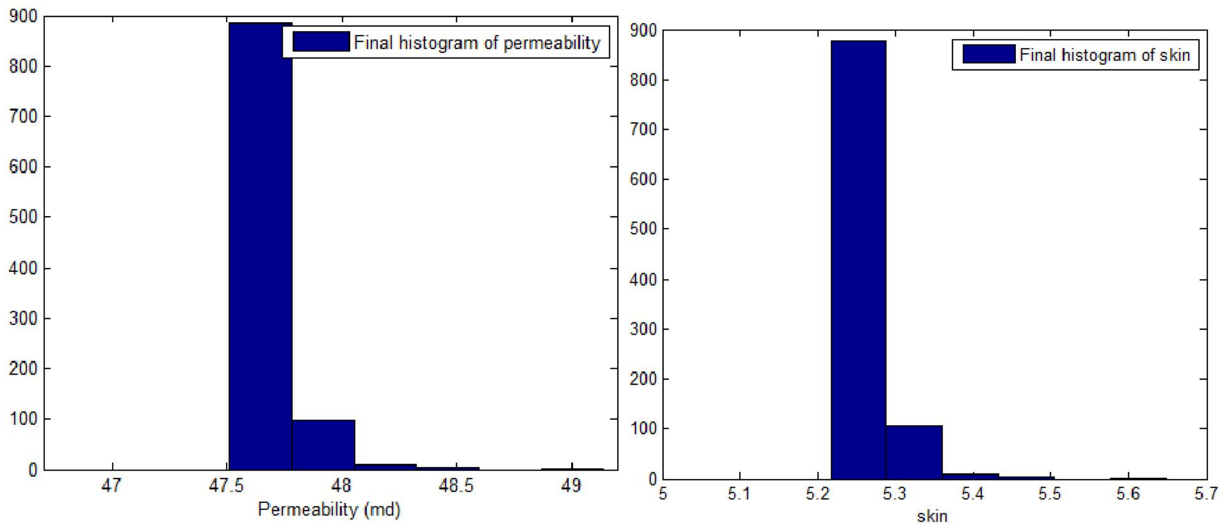


Figure 11: Historam of releazations of permeability (left) and skin factor (right) after assimilation final measurment.

Table 3: Summary of results from real well test data using ensemble Kaman filter

Unknown parameters	True value	Mean final	range	R_k	STD
Wellbore storage coefficient (bbl/psi)	0.023	0.022845	[0.02-0.025]	1	0.00071
Permeability (md)	48	47.8638	[47.5-48.5]	2	0.44413
Skin factor	5.37	5.2788	[5.2-5.5]	2	0.11762

4. Discussion

We found that the EnKF is suitable for data from time series when the changes made to the model parameters and state variables are both small at every measurement time. However, when the changes in the variables are large or numbers of variable is high, the EnKF may provide invalid solutions because only pressure versus time is the observation vector and it is not usually enough. We need other observation vectors to decrease the effect of bottom-hole pressure.

The performance of the EnKF depends on the number of realizations and distribution of these realizations at the first time step.

The noise is other important case in the algorithm that contains observational data, which is expressed by R_k. This R_k is standard deviation of added noise to the observation data and must change by the time steps, but we assumed it constant.

The results with a relatively small number of ensemble models are remarkably good. It seems that larger ensemble will be required for problems with larger amounts of data to be assimilated. At this time, we do not know if the results might begin to deteriorate if the assimilation period is much longer or if the models are much larger. We also find from both examples that with the arrival of significant data at some measurement times, the error in the estimates of variables may grow after assimilating the data. This might be caused by the big changes in the state variables where the formulation used in the EnKF update formula is not acceptable.

History matching of petroleum reservoirs is a difficult task, and is very CPU demanding because of the reservoir simulator, which must be run a large number of times. Use of the Ensemble Kalman filter for history matching allows for parallel reservoir simulator runs, which save time. The Ensemble Kalman filter is tested and further developed for history matching by several people, and the results are promising. But still, much work can be done in this field.

5. Conclutions

We have demonstrated that the ensemble Kalman filter is an algorithm that is well suited for producing forecasts with uncertainty. It is observed that the forecasts are improved after assimilation of production data. For the first time, ensemble Kalman filter is used to identify unknown parameters and region time from the well test data.

The data assimilation algorithms were tested using synthetic reservoirs. Data was generated with reservoir models and noise was added. Real reservoir models which consists of a lot of noise and missed data are used to reconstruct the original parameters. We have shown that the ensemble mean of estimated static variables (the ensemble mean of permeability, skin factor and wellbore storage coefficient converge to the true values just after arriving a few points.

The results have been associated with the histograms incorporating the range of value with a specified probability, we have concluded that any value of final histogram could be a result, but with a specified probability. While in conventional method a constant value is represented a parameter.

We have observed in the synthetic and real well test when as we were calculating the two parameters simultaneously, their behavior is similar at each time step. This means that if permeability changes in every time step, skin changes at the same time step.

6. Nomenclature

E_i	: Exponential integral
EKF	: Extended Kalman Filter
EnKF	: Ensemble Kalman Filter
F_t	: Nonlinear relationship
H	: Measurement matrix
i	: Index (members of ensemble)
K	: Kalman gain matrix
Ne	: Number ensemble members
P	: Covariance matrix of model uncertainty
L	: Difference matrix
x	: State vector
y	: Measurement data
<Greek Symbols >	
θ	: System parameter vector
w_t	: Additive Gaussian noise
${\sigma_w}^2$	: Variance additive Gaussian noise
μ	: Viscosity
v_t	: Additive Gaussian noise
${\sigma_v}^2$	: Variance additive Gaussian noise
<Subscripts>	
t	: Time step index
k	: Time step index
<Superscripts >	
u	: updated state, updated, meaning that the values are from the assimilation step
p	: Predicted state, prior, meaning that the values are from the forward step
T	: Transpose

7. References

[1]. Bertino, L., G. Evensen, and H. Wackernagel, Sequential data assimilation techniques in oceanography, International Statistical Review, 71(2), 223–241, 2003.

[2]. Bourdet, D., Well test analysis: the use of advanced interpretation models, Elsevier Amsterdam, 2002.

[3]. Corser, G. P., J. E. Harmse, B. A. Corser, M. W. Weiss, and G. L. Whitflow, Field test results for a real-time intelligent drilling monitor, SPE-59227, in Proceedings of the 2000 IADC/SPE Drilling Conference, 2000.

[4]. Eisenmann, P., M.-T. Gounot, B. Juchereau, and S. J. Whittaker, Improved rxo measurements through semi-active focusing, SPE-28437, in Proceedings of the SPE 69th Annual Technical Conference and Exhibition, 1994.

[5]. Evensen, G., Sequential data assimilation with a non-linear quasi-geostrophic model using Monte Carlo methods to forecast error statistics. Journal of Geophysical Research, 99(C5):10143– 10162, 1994.

[6]. Evensen G., The ensemble Kalman filter: theoretical formulation and practical implementation, Ocean Dyn. 53, 343- 367, 2003.

[7]. Gu, Y. and D. S. Oliver, The ensemble Kalman filter for continuous updating of reservoir simulation models, Journal of Energy Resources Technology, 128(1), 79–87, 2006.

[8]. Horne, R. N., Modern Well Test Analysis: A Computer-Aided Approach, Petro Way; 2nd edition, 1995

[9]. Houtekamer P.L., and H.L. Mitchell, Data assimilation using an Ensemble Kalman Filter technique, Mon Weather Rev 126: 796-811, 1998.

[10]. Jazwinski A.H., Stochastic processes and filtering theory, Academic, San Diego, California, 1970.
[11]. Kalman R.E., A New Approach to Linear Filtering and Prediction Problems, Transactions of the ASME--Journal of Basic Engineering. vol. 82, pp. 35-45, 1960.
[12]. Lewis, M., Lakshmivarahan, S., Dynamic data assimilation a least squares approach, Cambridge University Press 2006.
[13]. Lorentzen R.J., Naevdal G., Lage A.C.V.M., Tuning of parameters in a two-phase flow model using an ensemble Kalman filter, International Journal of Multiphase Flow, 29, 1283-1309.2003.
[14]. Navdal G., L.M. Johnsen, S.I. Aanonsen, and E.H. Vefring, Reservoir monitoring and continuous model updating using ensemble Kalman filter, SPE- 84372, 2003.
[15]. Navdal G., T. Mannseth, and E.H. Vefring, Near-well reservoir monitoring through ensemble Kalman filter, SPE-75235, 2002.
[16]. Reichle R.H., D.B. McLaughlin, and D. Entekhabi, Hydrologic data assimilation with the Ensemble Kalman Filter, Mon Weather Rev 130: 103-114, 2002.
[17]. Vazquez, A. and Syversveen, The Ensemble Kalman Filter theory and applications in oil industry, Technical Report, Norwegian Computing Center, 2006.
[18]. Zang, X. and P. Malanotte-Rizzoli, A comparison of assimilation results from the ensemble Kalman filter and a reduced-rank extended Kalman filter, Nonlinear Processes in Geophysics, 10(6), 477–491, 2003.

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