Laplace Transformation of Infinite Series

Dr. Dinesh Verma

Associate Professor, Yogananda College of Engineering & Technology, Jammu
Email: drdinesh.maths@gmail.com

Abstract: Generally it has been noticed that differential equation is solved typically. The Laplace transformation makes it easy to solve. The Laplace transformation is applied in different areas of science, engineering and technology. The Laplace transformation is applicable in so many fields. Laplace transformation is used in solving the time domain function by converting it into frequency domain. Laplace transformation makes it easier to solve the problems in engineering applications and makes differential equations simple to solve. In Mathematics, a power series in one variable is an infinite series. In this paper we will discuss the Laplace Transformation of some infinite series.

[Dinesh Verma. **Laplace Transformation of Infinite Series.** *J Am Sci* 2019;15(6):56-59]. ISSN 1545-1003 (print); ISSN 2375-7264 (online). http://www.jofamericanscience.org. 8. doi:10.7537/marsjas150619.08.

Key words: Laplace transformation, Inverse Laplace transformation, Power Series.
Sub area: Laplace transformation
Broad area: Mathematics

Introduction:

Laplace transformation is a mathematical tool which is used in the solving of differential equations by converting it from one form into another form. Generally it is effective in solving.

linear differential equation either ordinary or partial. It reduces an ordinary differential equation into algebraic equation. Ordinary linear differential equation with constant coefficient and variable coefficient can be easily solved by the Laplace transformation method without finding the generally solution and the arbitrary constant. It is used in solving physical problems. this involving integral and ordinary differential equation with constant and variable coefficient.

It is also used to convert the signal system in frequency domain for solving it on a simple and easy way. It has some applications in nearly all engineering disciplines, like System Modeling, Analysis of Electrical Circuit, Digital Signal Processing, Nuclear Physics, Process Controls, and Applications in Probability, Applications in Physics, and Applications in Power Systems Load Frequency Control etc.

Definition

Let F (t) is a well defined function of t for all t ≥ 0. The Laplace transformation of F (t), denoted by f (p) or L {F (t)}, is defined as

L {F (t)} $=\int_0^\infty e^{-pt}F(t)dt = f(p)$

Provided that the integral exists, i.e. convergent. If the integral is convergent for some value of p, then the Laplace transformation of F (t) exists otherwise not. Where p the parameter which may be real or complex number and L is is the Laplace transformation operator.

The Laplace transformation of F (t) i.e. $\int_0^\infty e^{-pt}F(t)dt$ exists for p>a, if

F (t) is continuous and $\lim_{n\to\infty}\{e^{-at}F(t)\}$ is finite. It should however, be keep in mind that above condition are sufficient and not necessary.

Laplace transformation of Convergence Series:

$$\frac{c+z}{1!}+\frac{(c+2z)^2}{2!}+\frac{(c+3z)^3}{3!}+\cdots$$

$$=\sum_{n=0}^{\infty}\frac{(c+nz)^n}{n!}=\mathrm{f}\,(\mathrm{z})$$

$$\text{So, } \mathrm{L}\{\mathrm{f}(\mathrm{z})\}=\mathrm{L}\left\{\sum_{n=0}^{\infty}\frac{(c+nz)^n}{n!}\right\}$$

$$dz, let\ c+nz=t$$

$$=\sum_{n=0}^{\infty}\int_0^\infty e^{-pz}\frac{(c+nz)^n}{n!}dz$$

$$=\sum_{n=0}^{\infty}\int_0^\infty e^{-p\left(\frac{t-c}{n}\right)}\frac{t^n}{n!}\frac{dt}{n}$$

$$=\sum_{n=0}^{\infty}e^{\frac{pc}{n}}\int_0^\infty e^{-p\left(\frac{t}{n}\right)}\frac{t^n}{n!}\frac{dt}{n}, let\ \frac{t}{n}=u$$

$$=\sum_{n=0}^{\infty}e^{\frac{pc}{n}}\int_0^\infty e^{-pu}\frac{n^n u^n}{n!}\frac{ndu}{n}$$

$$=\sum_{n=0}^{\infty}e^{\frac{pc}{n}}\frac{n^n}{n!}\int_0^\infty e^{-pu}u^n du$$

$$=\sum_{n=0}^{\infty}e^{\frac{pc}{n}}\frac{n^n}{n!}\frac{n!}{p^{n+1}}$$

Hence,

$$L\left\{\sum_{n=0}^{\infty}\frac{(c+nz)^n}{n!}\right\}=\sum_{n=0}^{\infty}e^{\frac{pc}{n}}\frac{n^n}{p^{n+1}}$$

Laplace Transformation of Geometric Series:

$$\frac{1}{1-z}=\sum_{n=0}^{\infty}z^n=f(z)$$

$$L\{f(z)\}=L\left\{\sum_{n=0}^{\infty}z^n\right\}$$

$$=\int_0^{\infty}e^{-pz}\sum_{n=0}^{\infty}z^n\,dz$$

$$=\sum_{n=0}^{\infty}\int_0^{\infty}e^{-pz}z^n dz$$

$$=\sum_{n=0}^{\infty}L\{z^n\}$$

$$=\sum_{n=0}^{\infty}\frac{n!}{p^{n+1}}$$

Hence,

$$L\{f(z)\}=\sum_{n=0}^{\infty}\frac{n!}{p^{n+1}}$$

Laplace Transformation of the Power series expansion of e^z:

$$e^z=\sum_{n=0}^{\infty}\frac{z^n}{n!}=f(z)$$

$$L\{f(z)\}=L\left\{\sum_{n=0}^{\infty}\frac{z^n}{n!}\right\}$$

$$=\int_0^{\infty}e^{-pz}\left\{\sum_{n=0}^{\infty}\frac{z^n}{n!}\right\}dz$$

$$=\sum_{n=0}^{\infty}\int_0^{\infty}e^{-pz}\frac{z^n}{n!}dz$$

$$=\sum_{n=0}^{\infty}\frac{1}{n!}\int_0^{\infty}e^{-pz}z^n dz$$

$$=\sum_{n=0}^{\infty}\frac{1}{n!}L\{z^n\}=\sum_{n=0}^{\infty}\frac{1}{p^{n+1}}$$

$$Hence, L\{f(z)\}=\sum_{n=0}^{\infty}\frac{1}{p^{n+1}}$$

Laplace Transformation of the Power series expansion of $log(1+z)$:

$$\log(1+z)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}z^n=f(z)$$

$$L\{f(z)\}=L\left\{\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}z^n\right\}$$

$$=\int_0^{\infty}e^{-pz}\left\{\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}z^n\right\}dz$$

$$=\sum_{n=1}^{\infty}\int_0^{\infty}e^{-pz}\frac{(-1)^{n+1}}{n}z^n dz$$

$$=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\int_0^{\infty}e^{-pz}z^n dz$$

$$=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}L\{z^n\}$$

$$=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\frac{n!}{p^{n+1}}$$

$$=\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(n-1)!}{p^{n+1}}$$

Hence,

$$L\{f(z)\}=\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(n-1)!}{p^{n+1}}$$

Laplace Transformation of the Power series expansion of $log(1+z)$:

$$\log(1+z)=\sum_{n=1}^{\infty}\frac{(-1)^{2n-1}}{n}z^n=f(z)$$

$$L\{f(z)\}=L\left\{\sum_{n=1}^{\infty}\frac{(-1)^{2n-1}}{n}z^n\right\}$$

$$=\int_0^{\infty}e^{-pz}\left\{\sum_{n=1}^{\infty}\frac{(-1)^{2n-1}}{n}z^n\right\}dz$$

$$=\sum_{n=1}^{\infty}\int_0^{\infty}e^{-pz}\frac{(-1)^{2n-1}}{n}z^n dz$$

$$=\sum_{n=1}^{\infty}\frac{(-1)^{2n-1}}{n}\int_0^{\infty}e^{-pz}z^n dz$$

$$=\sum_{n=1}^{\infty}\frac{(-1)^{2n-1}}{n}L\{z^n\}$$

$$=\sum_{n=1}^{\infty}\frac{(-1)^{2n-1}}{n}\frac{n!}{p^{n+1}}$$

$$=\sum_{n=1}^{\infty}(-1)^{2n-1}\frac{(n-1)!}{p^{n+1}}$$

Hence,

$$L\{f(z)\} = \sum_{n=1}^{\infty} (-1)^{2n-1} \frac{(n-1)!}{p^{n+1}}$$

Laplace Transformation of the Power series expansion of $\log\frac{(1+z)}{(1-z)}$**:**

$$\log\frac{(1+z)}{(1-z)} = \sum_{n=1}^{\infty} \frac{2}{2n-1} z^{2n-1} = f(z)$$

$$L\{f(z)\} = L\left\{\sum_{n=1}^{\infty} \frac{2}{2n-1} z^{2n-1}\right\}$$

$$let,\ 2n-1 = m$$

$$2n = m+1$$

$$n = \frac{m+1}{2}$$

$$= \int_0^{\infty} e^{-pz}\left\{\sum_{n=1}^{\infty} \frac{2}{2n-1} z^{2n-1}\right\} dz$$

$$= \sum_{n=1}^{\infty} \int_0^{\infty} e^{-pz} \frac{2}{2n-1} z^{2n-1} dz$$

$$= \sum_{n=1}^{\infty} \frac{2}{2n-1} \int_0^{\infty} e^{-pz} z^{2n-1} dz$$

$$= \sum_{n=1}^{\infty} \frac{2}{2n-1} L\{z^{2n-1}\}$$

$$\sum_{n=1}^{\infty} \frac{2}{2n-1} \frac{(2n-1)!}{p^{2n-1+1}}$$

$$\sum_{n=1}^{\infty} \frac{(2n-1)!}{p^{2n}}$$

Hence,

$$L\{f(z)\} = \sum_{n=1}^{\infty} \frac{(2n-1)!}{p^{2n}}$$

Laplace Transformation of the Power series expansion of $Cosx$**:**

$$Cosx = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n!} z^{2n} = f(z)$$

$$L\{F(z)\} = L\left\{\sum_{n=0}^{\infty} \frac{(-1)^n}{2n!} z^{2n}\right\}$$

$$= \int_0^{\infty} e^{-pz}\left\{\sum_{n=0}^{\infty} \frac{(-1)^n}{2n!} z^{2n}\right\} dz$$

$$= \sum_{n=0}^{\infty} \int_0^{\infty} e^{-pz} \frac{(-1)^n}{2n!} z^{2n} dz$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{2n!} \int_0^{\infty} e^{-pz} z^{2n} dz$$

$$let\ 2n = u$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{2n!} L\{z^u\}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{2n!} \frac{u!}{p^{u+1}}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{2n!} \frac{2n!}{p^{2n+1}}$$

$$Hence,\ L\{f(t)\} = \sum_{n=0}^{\infty} \frac{(-1)^n}{p^{2n+1}}$$

Laplace Transformation of the Power series expansion of $Sinx$**:**

$$Sinx = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} z^{2n+1} = f(z)$$

$$L\{F(z)\} = L\left\{\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} z^{2n+1}\right\}$$

$$= \int_0^{\infty} e^{-pz}\left\{\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} z^{2n+1}\right\} dz$$

$$= \sum_{n=0}^{\infty} \int_0^{\infty} e^{-pz} \frac{(-1)^n}{(2n+1)!} z^{2n+1} dz$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \int_0^{\infty} e^{-pz} z^{2n+1} dz$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} L\{z^{2n+1}\}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{(2n+1)!}{p^{2n+1+1}}$$

$$Hence,\ L\{f(t)\} = \sum_{n=0}^{\infty} \frac{(-1)^n}{p^{2n+2}}$$

Laplace Transformation of the Power series expansion of $Coshx$**:**

$$Cosx = \sum_{n=0}^{\infty} \frac{1}{2n!} z^{2n} = f(z)$$

$$L\{F(z)\} = L\left\{\sum_{n=0}^{\infty} \frac{1}{2n!} z^{2n}\right\}$$

$$= \int_0^{\infty} e^{-pz}\left\{\sum_{n=0}^{\infty} \frac{1}{2n!} z^{2n}\right\} dz$$

$$= \sum_{n=0}^{\infty} \int_0^{\infty} e^{-pz} \frac{1}{2n!} z^{2n} dz$$

$$= \sum_{n=0}^{\infty} \frac{1}{2n!} \int_0^{\infty} e^{-pz} z^{2n} dz$$

$$let\ 2n = u$$

$$= \sum_{n=0}^{\infty} \frac{1}{2n!} L\{z^u\}$$

$$= \sum_{n=0}^{\infty} \frac{1}{2n!} \frac{u!}{p^{u+1}}$$

$$= \sum_{n=0}^{\infty} \frac{1}{2n!} \frac{2n!}{p^{2n+1}}$$

$$Hence, L\{f(t)\} = \sum_{n=0}^{\infty} \frac{1}{p^{2n+1}}$$

Laplace Transformation of the Power series expansion of $Sinx$:

$$Sinhx = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} z^{2n+1} = f(z)$$

$$L\{F(z)\} = L\left\{\sum_{n=0}^{\infty} \frac{1}{(2n+1)!} z^{2n+1}\right\}$$

$$= \int_0^{\infty} e^{-pz} \left\{\sum_{n=0}^{\infty} \frac{1}{(2n+1)!} z^{2n+1}\right\} dz$$

$$= \sum_{n=0}^{\infty} \int_0^{\infty} e^{-pz} \frac{1}{(2n+1)!} z^{2n+1} dz$$

$$= \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} \int_0^{\infty} e^{-pz} z^{2n+1} dz$$

$$= \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} L\{z^{2n+1}\}$$

$$= \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} \frac{(2n+1)!}{p^{2n+1+1}}$$

$$Hence, L\{f(t)\} = \sum_{n=0}^{\infty} \frac{1}{p^{2n+2}}$$

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6/17/2019