Abstract: Image reconstruction in computed tomography (CT) has been extensively studied from different angles, to find new methods and algorithms for better execution of the reconstruction tasks. In this study, three challenging problems in developing a prototype for the image reconstruction system using a summation back projection approach are highlighted namely; blurring effect of the summation back projection computational processes, computational complexity and the necessity for a graphical user interface. Our proposed solutions, which respectively address these problems are to exploit Ram-Lak filter, to utilize a scientific and industrial programming language (FORTRAN) and to use Visual Basic Studio (VB). To examine the solutions, a prototype software has been developed, by which sinograms and slice form of an object’s image can be generated. Furthermore, this article presents a comprehensive background study on image reconstruction using filtered back projection (FBP) approaches to develop the CT scanner software application.

Keywords: Tomography; non-destructive testing; Radon Transform; Fourier Slice Theorem; sinogram; back projection; image reconstruction; filtered; fan-beam; convolution; attenuation coefficient

1. Introduction

The technique of non-destructive method to observe the inside property of an object was established as early as 1935. Recently, people have recognized tomography as CT scans; the two letters CT are the acronym used for computed tomography. CT scans technologies have evolved from the first generation to the existing fifth generation. The organization that provides a non-destructive testing (NDT) training course in Malaysia is the Malaysian Nuclear Agency. The purpose of this article is to propose an NDT software for the industrial division at Malaysian Nuclear Agency.

2. Material and Methods

The word tomography originates from two Greek words which are ‘tomes’ and ‘graph’. The word ‘tomes’ means slice while ‘graph’ means picture (Rahim, 2005), thus the word tomography means a slice of a picture. Tomography in other words can be defined as the cross sectional image of an object. The CT scanner is known to be the tool of non-destructive method to view the internal properties of the object. Table 1 shows the generations of the CT scanner.

<table>
<thead>
<tr>
<th>Scanner Generation</th>
<th>Scanner Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>First-generation</td>
<td>The first-generation scanner is the combination of linear translation followed by incremental rotation, and is well-known as translate-rotate motion (Young et al., 1999; Goldman, 2007).</td>
</tr>
<tr>
<td>Second-generation</td>
<td>The second-generation CT uses multiple narrow beams from a single source and multiple detectors. The scanner movement is called rotate-translate (Young et al., 1999; Goldman, 2007).</td>
</tr>
<tr>
<td>Third-generation</td>
<td>The third generation CT scanner detector array is rigidly linked to the X-ray tube, so that both the tube and the detectors are rotated together around the patient (a motion is referred to as a rotation-rotation) (Young et al., 1999; Goldman, 2007).</td>
</tr>
<tr>
<td>Fourth-generation</td>
<td>The fourth-generation CT scanner is incorporated with a large stationary ring of detectors, with the X-ray tube alone rotating around the patient (Goldman, 2007).</td>
</tr>
<tr>
<td>Fifth-generation</td>
<td>The fifth-generation CT scanner refers to a multi-slice CT scanner that is equipped with a multiple-row detector array (Young et al., 1999).</td>
</tr>
</tbody>
</table>
Data Acquisition and Image Reconstruction

In general, the process of developing a CT scanner application software within this research area can be divided into two different main processes: (1) data acquisition and (2) image reconstruction. In the earliest CT imaging technology, data acquisition device is a narrow X-ray beam scanned across the object in synchronization with a radiation detector on the opposite side of the object. The radiation absorption is governed by the Beer Lambert law, also alternatively known as the Beer's law as in Equation 1 (Kimia, 2011).

\[ I = I_0 e^{-\mu x} \]  

(1)

The projection of the datasets may be acquired by one of possible geometries as described in Table 1 (Cunningham & Judy, 2000). This datasets acquisition is based on the scanning configuration, scanning motions and detector arrangement. The images from a CT scanner are divided into two general categories, which are sinogram image and object slice image.

a) Sinogram Image

A sinogram is the collection of parallel projections of the object taken at equidistant angles, and forming a map of the projection data (Azevedo, et al., 2001). Dove (2001) stated that stacking all the projections \( p_\theta(r) \) together resulted in a 2D data set \( p(r, \theta) \), which is also known as a sinogram. A sinogram based on the concept of line integral and Radon transform is used for image reconstruction processes. A sinogram is capable of explaining the Radon transform and line integral in producing the image of an object (Zakaria et al., 2010).

An Austrian mathematician, named Johann Radon, introduced the Radon Transform and whom also founded tomography reconstruction. The mathematical reconstruction of the projections obtained from a CT scanner is the conversion of a function \( f(x, y) \) into \( p(r, \theta) \). Further, 2-D section images could be reformulated using mathematical transformation of projection data (Lancaster, 2012). Projection data are line integrals recorded across an object at some angle. The Radon Transform can be defined as the collection of projections of an object gathered from different angles (Ramotar & Orchard, 2006). Equation 2 shows the function of \( f(x, y) \) (Toft, 1996).

\[ h(\rho, \theta) = \iint_{-\infty}^{\infty} f(x, y) \delta(\rho - x\cos \theta - y\sin \theta) \, dx \, dy \]  

(2)

b) Object Slice Image

The CT image reconstruction from projections is the process of producing an image of the two-dimensional distribution of some physical property from estimates of its line integrals along a finite number of lines of known locations (Yazgan et al., 1992).

Reconstruction Algorithms

Image reconstruction is the process of estimating an object image slice of \( f(x, y) \) from a set of projections \( p(r, \theta) \). Several algorithms with different advantages can accomplish this task. The foundation of the mathematical package for image reconstruction is the reconstruction algorithm, which may be one of four types (Hendee & Ritenour, 2002). The fundamental reconstruction algorithms are:

a) Simple Back projection

The algorithm of back projection approach to reconstruct an image is straightforward, but produces a blurry image feature of the object.

b) Filtered Back projection

The FBP algorithm is often referred as the convolution method using a one-dimensional integral equation for the reconstruction of a two-dimensional image. The convolution method is the most popular reconstruction algorithm used at present in CT application.

c) Fourier Transform

The attenuation patterns at each angular orientation are separated into frequency components of various amplitudes, then transformed into a spatially correct image, and then reconstructed by an inverse Fourier transform process.

d) Series of Mathematical Expansion

The variations of mathematical expansion are known as ART (algebraic reconstruction technique), ILST (iterative least-squares technique), and SIRT (simultaneous iterative reconstruction technique).
Research Methodology

The main goal of this research is to study an image reconstruction technique for CT scanner and its analysis. This study also aims to propose a new technique for faster reconstruction time as well as to develop a prototype for image reconstruction. Additionally, a Rapid Application Development (RAD) model was used as the main method in the prototype development process. The development of the research is divided into four phases.

Phase 1: Preliminary

The preliminary study was conducted at the Malaysian Nuclear Agency, Bangi, Selangor. The study of image reconstruction techniques was carried out with respects to the first and the third-generation CT scanner. Due to the limitation of the prototype program which is written in a Visual Basic language, large data set of projection and ray sums could not be successfully reconstructed.

The data set of tester plate was used to test the efficiency of the proposed algorithms. The Visual Basic was selected as the programming language due to its suitability in developing a user interface and to compile the codes written for the proposed system. In addition, other programming languages including FORTRAN and C++ were selected for the computation purpose. Figure 2 shows a preliminary CT scanner imaging prototype at the research center.

Phase 2: Design and Development of Prototypes

The development of the prototype comprised of three stages: 1) the user requirement analysis, 2) the interface design and 3) the programming stages. In the first stage, the literature search and analysis was conducted to obtain information regarding the user requirement on the detailed parallel-beam geometry in the CT scanner. During the second stage, the human and machine interfaces were designed to allow users to communicate with the imaging system. In the third stage, the selected programming languages such as Visual Basic, FORTRAN and C++ were used to code the major computational functions. The processes of developing the prototype were divided into sinogram and summation back projection.

a) Sinogram

Plotting a sinogram from a collection of datasets is a preliminary step towards a reconstruction process to obtain a slice of the image of the object. A sinogram is able to give a general view of information about datasets being processed and thus gives an early warning of data error.

b) Summation Back projection

Summation back projection approach is among the well-known algorithms in the area of image reconstruction. The advantages of back projection method in the image reconstruction process relate to its ability of performing as a real time imaging technique. However, this method produced a blurring image after completion of the last projection of reconstruction. Figure 3 shows a flowchart of the proposed system for the reconstructions of the image for the CT scanner.

Ram-Lak Filter

The standard programming syntax for Visual Basic and C++ language are similar where an array counting starts from zero, however, the FORTRAN programming language defines an array to start from one. The knowledge to integrate both of the languages is required in completing the filtering task. Ramachandran & Lakshminarayanan (1971) proposed the back projection method to overcome the
blurring image which is later known as Ram-Lak filter as shown in Equation 3.

\[ h_{RL}(t) = 2B^2 \text{Sinc}(2tB) - B^2 \text{Sinc}^2(tB) \]  

(3)

where, \( B = 1/2d \) and \( \text{Sinc}(x) = \sin(x)/x \). The discrete expression is shown in Equation 4 (Wang et al., 2010).

\[ h_{RL}(nd) = \begin{cases} \frac{1}{n^2d^2} & n = 0 \\ 0 & n = \text{even} \\ -\frac{1}{n^2\pi^2d^2} & n = \text{odd} \end{cases} \]  

(4)

The program pseudo code 1 and 2 in Basic language shows the Ram-Lak filter aims to solve the blurring effect when using the summation back projection method in constructing the image. The filter algorithm is in one-dimensional array, and the object datasets are in two-dimensional array. The programming task is to cross-correlation the filter datasets and the object datasets. The array size is the total number of datasets in one projection. Moreover, an array size is an important variable as it is required before the compilation could successfully be initiated. The program pseudo code 1 uses FOR statement in assigning the value zero into the only even array pointer.

Program Pseudo code 1

\[
h_{od}= \text{half} \_\text{of} \_\text{dataset} \\
\text{FOR} \ m = -h_{od} \text{ TO hod STEP} 2 \\
\ h(m + hod) = 0 \\
\text{NEXT} \ m
\]

The program pseudo code 2 uses FOR statement in assigning the Ram-Lak values into the odd array pointer and to complement the array operations.

Program Pseudo code 2

\[
h_{od}= \text{half} \_\text{of} \_\text{dataset} \\
\text{FOR} \ m = -h_{od} + 1 \text{ TO hod - 1 STEP} 2 \\
\ h(m + hod) = -1 / (m \ast m) \\
\text{NEXT} \ m \\
\ h(hod) = (\pi^2/4)
\]

**Convolution**

Convolution is the action of multiplying two input signal functions to produce a single output product, and was carried out between datasets obtained from the data acquisition system and datasets generated by Ram-Lak filter. The goal of convolution is to remove the blurring effect caused by the summation back projection. The computational complexity of a convolution routine is \( O(n^2) \), therefore, in the case of CT datasets, a scientific and industrial compiler is required to perform the computational tasks. The program pseudo code 3 shows three FOR loop statements to convolve the object datasets with the filter datasets. The outer loop represents a number of projections and the two inner loops represent two-dimensional datasets.

Program Pseudo code 3

\[
h_{od}= \text{half} \_\text{of} \_\text{dataset} \\
\text{nop}\_\text{number} \_\text{of} \_\text{projection} \\
\text{FOR} \ q = 0 \text{ TO} \ \text{nop} - 1 \\
\ \text{FOR} \ mx = -h_{od} \text{ TO} \ hod \\
\ \text{FOR} \ mm = -h_{od} \text{ TO} \ hod \\
\ \ \text{raysum} = \text{ray}(mm + hod, q) \\
\ \ \text{summ} = \text{summ} + \text{raysum} \ast h(m1 + m2) \\
\ \text{NEXT} \ mm \\
\ \text{temp}(mx + hod) = \text{summ} \\
\ \text{NEXT} \ mx \\
\ \text{FOR} \ m = -h_{od} \text{ TO} \ hod \\
\ \ \text{raysum}(m + hod, q) = \text{temp}(m + hod) \\
\ \text{NEXT} \ m \\
\text{NEXT} \ q
\]

**Image Reconstruction**

The image reconstruction boundary is defined by a circle where a radius of a circle is a half of datasets of one projection. Each projection was rotated in a circle at a constant interval of a different angle. The CT scanner motion could be represented by a transformation of a rotation. The matrix algebra in Equation 5 shows a rotation transformation of a matrix \((x, y)\) into matrix \((r, s)\).

\[
\begin{bmatrix} r \\ s \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}
\]  

(5)

where \( r = x\cos(\theta) - y\sin(\theta) \) and \( s = x\sin(\theta) + y\cos(\theta) \)

The program pseudo code 4 shows three FOR statements in constructing the image. The outer loop represents a number of projections and the two inner loops represent a point \((x, y)\). A two-dimensional array was used to accumulate the summation value of datasets.

Program Pseudo code 4

\[
\text{nop} = \text{number} \_\text{of} \_\text{projection} \\
\text{for} \ h_{od} = \text{half} \_\text{of} \_\text{dataset}
\]
The array of datasets from the image reconstruction routine can be plotted as a graphical image on the computer screen. The dynamic graphic resolution of height and width of image pixel size can be changed using variable ‘imageSize’ and the datasets are required to be normalized by using a grayscale level. Lastly, the picture-box height and width are required to be normalized in order to plot the image. The program pseudo code 5 shows two FOR statements to plot the datasets obtained from the image reconstruction computational procedure.

Program Pseudo code 5

```
imageSize = 100

dwp = imageSize / (dataset per projection)
dwx = imageSize / (dataset per projection)

FOR j = 0 TO imageSize - 1
   ryap = INT(j / dwp)
   FOR i = 0 TO 80 - 1
      rxap = INT(i / dwx)
      gsv = p(ryap, rxap)
      PictureBox(j, i), RGB(gsv, gsv, gsv)
   NEXT i

NEXT j
```

Phase 4: Analyses and Evaluation

Figure 4(b) and Figure 5(b) shows the result from the testing phase which displays the total number of projections that controls the quality of the image generated by the prototype application. In addition, Table 2 shows the time consumed by FORTRAN and Visual Basic subroutine in completing the image reconstruction routine. The testing was carried out using Intel(R) Core(TM) i5-2410M CPU @2.30GHz, 64-bit operating system and 4GB of computer memory. The findings showed that the FORTRAN library is better in comparison to Visual Basic library to be utilized for CT scanner computational application. The results obtained in this study are expected to provide useful information for future development of the CT scanner application.

Table 2: The time taken by FORTRAN and Visual Basic

<table>
<thead>
<tr>
<th>Sample of Datasets</th>
<th>FORTRAN</th>
<th>VB</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 projections 59 ray sums</td>
<td>1 sec</td>
<td>1 sec</td>
</tr>
<tr>
<td>240 projections 171 ray sums</td>
<td>3 secs</td>
<td>23 secs</td>
</tr>
</tbody>
</table>

Figure 4: Sinogram and dot image from 4 projections and 59 ray sums

Figure 5: Sinogram and dot image from 20 projections and 59 ray sums
3. Results and Discussion

The image obtained from the back projection construction without data filtering was blurry. Therefore, a filtering process is needed to remove the noise which blurs the image. The phenomenon occurs because of the summation back projection effect. Figure 6(a) and Figure 6(b) shows a test result obtained from unfiltered data set of 240 projections. Meanwhile, Figure 7(a) and Figure 7(b) shows a test result obtained from filtered data set of 240 projections.

The process of filtering was performed before the reconstruction process of the image took place. The filtering is the process of removing noise from the object data set using convolution technique. Moreover, it is realized that the image reconstructed by the filtered back projection algorithm is more accurate than the image reconstructed by the back projection filtering algorithm (Zeng & Gullberg, 1994).

![Sinogram and unfiltered image from 240 projections and 171 ray sums](image1)

(a) Sinogram  
(b) Unfiltered Image  
Figure 6: Sinogram and unfiltered image from 240 projections and 171 ray sums

![Sinogram and filtered image from 240 projections and 171 ray sums](image2)

(a) Sinogram  
(b) Filtered Image  
Figure 7: Sinogram and filtered image from 240 projections and 171 ray sums

4. Conclusion

In summary, the article has successfully demonstrated the back projection method in constructing images and discussed the technique of using two high-level languages to complement each other’s programming weaknesses. The proposed method has reduced the computation time and reconstructed projections and ray sums to produce a high-resolution image slice. The back projection approach has been proposed for the parallel beam data acquisition system. Further work is needed to transform the divergent beam datasets into parallel beam datasets and should take advantage of the method being discussed in this study. The research could extend to reconstruct images from the divergent beam geometry.

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